

1



# CHAPTER 1

## Problem Solving

Welcome to Algebra! What is algebra? This chapter will introduce you to many of the big ideas you will explore and the ways you will be working during this course. You will apply your current mathematical knowledge to solve problems, some of which you will revisit later in the course to solve using new algebraic tools.

This chapter will also introduce you to the five Ways of Thinking that are threaded throughout the course. They are: **justifying** (explaining and verifying your ideas), **generalizing** (predicting behavior for any situation), **making connections** (connecting your ideas to other ways of seeing or to past or future learning), **reversing thinking** (solving problems “backward and forward”), and **applying and extending** (applying your knowledge to new contexts and extending it to help solve new problems).

Finally, this chapter is about problem solving. During this chapter, you will use a variety of problem-solving strategies that will remain useful throughout this course, including:

Guessing and Checking

Collecting Data

Finding Patterns

Drawing a Graph

Working Backward

## Guiding Questions

Think about these questions throughout this chapter:

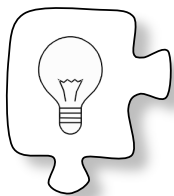
What is algebra?

How can I solve a problem that I have never seen before?

How can I organize my work?

How can I describe my process?

## Chapter Outline



**Section 1.1** This section will include several problems and activities that use many of the big ideas of algebra. Each one will require your study team to work together and use various problem-solving strategies.



**Section 1.2** This section introduces another problem-solving strategy that will help you solve a complex problem called “The Apartment.” You will also have opportunities to reflect on and write about your mathematical understanding by creating a Learning Log.

## 1.1.1 What stories can a graph tell?



### Interpreting Graphs

You will focus on several challenges during this unit that will require you to use different problem-solving strategies. While all of the problems are solvable with your current math skills, some will be revisited later in the course so that you can apply new algebraic tools to solve them. Each problem also introduces you to an important concept of algebra that you will study in this course.



#### 1-1. GETTING TO KNOW YOU, Part One

How can a graph tell a story? Today you will find your team members and then will work together to write a story for a graph.

**Your Task:** Your teacher will give you one part of a graph. Find the students in the class who have the other pieces of the same graph. When you find all of the students whose graph parts belong with yours, sit down together as a team.

As a team, come up with a story that could be represented by your team's graph. Think carefully about each part of the graph. Once your team agrees on a story, make sure every member of the team can describe each part of the story and explain its connection to the corresponding part of the graph.

To help you work together today, each member of the team has a specific job, assigned by your first name (or by your last name if any team members have the same first name).

## Team Roles

**Resource Manager** – If your name comes first alphabetically:

- Make sure that the team has tape.
- Ask the teacher when the **entire** team has a question. *“No one has an idea? Should I ask the teacher?”*
- Make sure your team cleans up by delegating tasks. You could say, *“I will put away the \_\_\_\_\_ while you \_\_\_\_\_.”*

**Facilitator** – If your name comes second alphabetically:

- Start the team’s discussion of the graph by asking, *“What could this graph be about?... What are some ideas?”*
- Help the team agree on a story: *“Do we agree on all of the parts of our story?”*

**Recorder/Reporter** – If your name comes third alphabetically:

- Tape the graph pieces together on a piece of paper to form the graph.
- Take notes for the team. The notes should include phrases like, *“For part one...”* and explanations like, *“Because part one is not so steep...”*

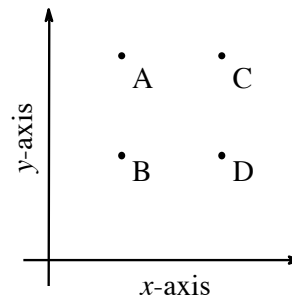
**Task Manager** – If your name comes fourth alphabetically:


- Remind the team to stay on task and not to talk to students in other teams. You can suggest, *“Let’s move on to another part of the graph.”*
- Listen for reasons and challenge your teammates to justify their thinking. *“But why do you think that?”*



1-2. GETTING TO KNOW YOU, Part Two

Suppose the graph at right represents something about the four students in your team. But what is the graph about? Decide what information the  $x$ - and  $y$ -axes could represent so that each point represents a different member of your team. **Justify** your statements.





MATH NOTES

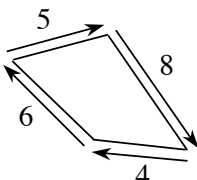
## METHODS AND MEANINGS

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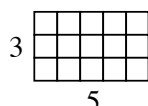
### The Perimeter and Area of a Figure

The **perimeter** of a figure is the distance around the exterior (outside) on a flat surface. It is the total length of the boundary that encloses the interior (inside) region. See the example at right.

The **area** indicates the number of square units needed to fill up a region on a flat surface. For a rectangle, the area is computed by multiplying its length and width. The rectangle at right has a length of 5 units and a width of 3 units, so the area of the rectangle is 15 square units.



Perimeter =  $5 + 8 + 4 + 6 = 23$  units



Area =  $5 \cdot 3 = 15$  square units



1-3. MATHOGRAPHY

Write a letter about yourself that will help your teacher get to know you as an individual, addressing each of the general topics below (in bold). Choose a few of the suggested questions to get you started.

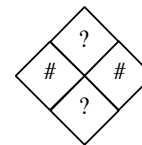
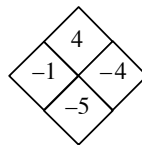
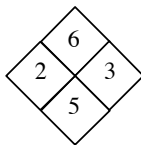
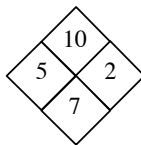
**About You:** By what name do you like to be called? What are your interests, talents, and hobbies? What are you proud of? With whom do you live? What languages do you speak? When is your birthday? What are you like as a member of a team? In what ways are you excited about working in a team? In what ways are you nervous about it?

**You as a Math Student:** Describe your memories as a math student from kindergarten until now. What experiences in math have you liked? Why? How do you feel about taking Algebra? Have you ever worked in a team in a math class before? What kinds of math do you imagine yourself doing in Algebra class?

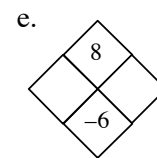
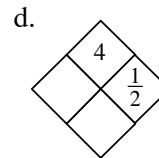
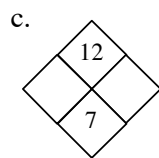
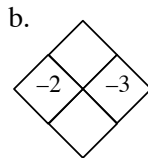
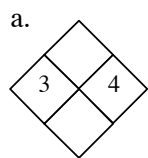
1-4. DIAMOND PROBLEMS

Finding and using a pattern is an important problem-solving skill you will use in algebra. The patterns in Diamond Problems will be used later in the course to solve other types of algebraic problems.

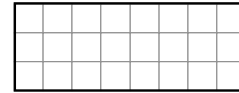
Look for a pattern in the first three diamonds below. For the fourth diamond, explain how you could find the missing numbers (?) if you know the two numbers (#).



Copy the Diamond Problems below onto your paper. Then use the pattern you discovered to complete each one.



- 1-5. The area of the rectangle at right is 24 square units. On graph paper, draw and label all possible rectangles with an area of 24 square units. Use only whole numbers for the dimensions (measurements).

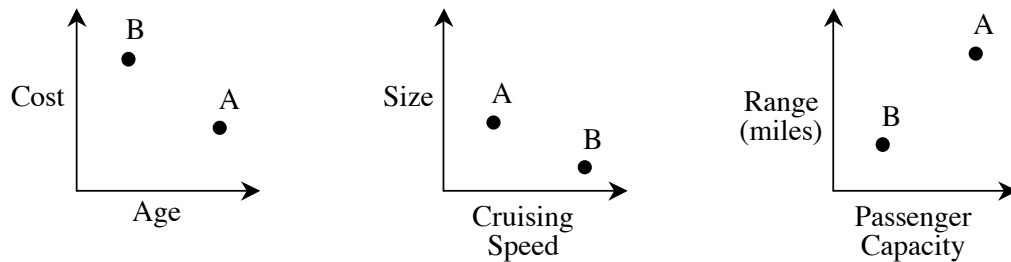


Area = 24 square units

- Find the perimeter of each of these rectangles. You may want to refer to the Math Notes box for this lesson for more information on the perimeter of a figure.
- Of these rectangles, which has the largest perimeter? Which has the smallest perimeter? Describe these shapes. Remember to use complete sentences.

1-6. CAR COMPARISON

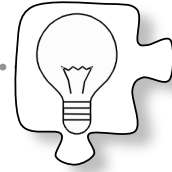
The following three graphs describe two cars, A and B.



Decide whether each of the following statements is true or false. Explain your reasoning.

- The newer car is more expensive.
- The slower car is larger.
- The larger car is newer.
- The cheaper car carries more passengers.

## 1.1.2 How can I name a point?



Using the  $(x, y)$  Coordinate Plane

Today you will use a **coordinate system** to refer to the locations of specific points in the form  $(x, y)$ . By the end of this lesson, be sure you and your teammates know the answers to the following target questions:

How can you plot a point using its  $(x, y)$  coordinates?

How can you name a point on the graph?

How can you describe a pattern formed by points?

### 1-7. THE EUCALYPTUS GROVE

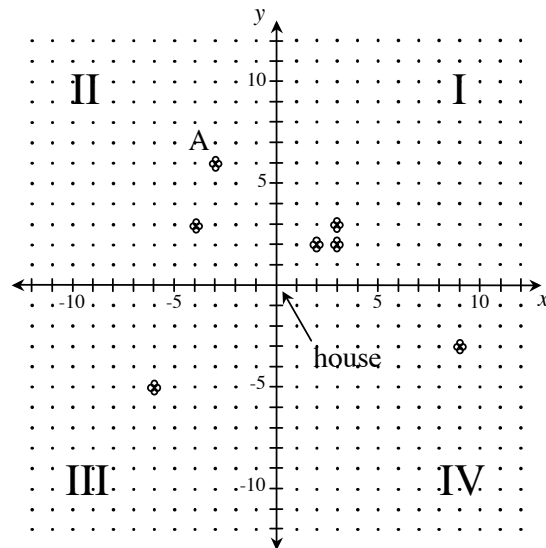
Some communities in Mozambique plant eucalyptus tree farms because their tall, straight trunks make great poles for building homes. While touring his farm, Etube (pronounced “eh-too-bay”) noticed that some of the trees have a deadly disease. He created a coordinate system, shown in the diagram below, to keep track of where the infected trees are. The farm’s roads are shown as the  $x$ - and  $y$ -axes in the diagram, and Etube’s house is located where the two roads intersect. Each section of the farm (known as a **quadrant**) has 12 rows of 12 trees. The trees marked with a  $\otimes$  show signs of disease – the tops are turning brown!



- a. The tree marked A, located at the point  $(-3, 6)$ , is diseased. In  $(x, y)$  form, list the locations of all of the other diseased trees shown on the diagram.

- b. On a piece of graph paper, neatly draw and label  $x$ - and  $y$ -axes to represent the roads intersecting at Etube’s house. Place points or symbols on your graph to represent the diseased trees.

Etube just learned that the trees at  $(-8, 2)$  and  $(5, -7)$  are also diseased. Add these trees to your diagram.



*Problem continues on next page →*

*Algebra Connections*



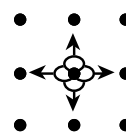
1-7. *Problem continued from previous page.*

- c. In  $(x, y)$  form, what is the location of Etube's house? This is also called the **origin** of a graph.
- d. Etube is standing at the farmhouse looking out at Quadrant I and sees that the tree at  $(3, 2)$  is diseased. However, he notices that he cannot see the tree at  $(6, 4)$  because it is blocked by the tree at  $(3, 2)$ .

In  $(x, y)$  form, write down the locations of two other trees that he cannot see from his house because they are hidden behind the tree at  $(3, 2)$ . What do the locations of these trees have in common? When looking at the coordinates, what pattern(s) do you notice?

- e. While standing at his house, can Etube see the diseased tree at  $(9, -3)$ ? If so, explain how you know. If not, name any trees that block its view. How can you be sure that these trees hide the tree at  $(9, -3)$ ? Be sure to **justify** your team's conclusion.

- f. The tree disease is passed each day when the leaves of a diseased tree touch the leaves of a healthy tree that grows next to it, as shown on the diagram at right. (Note that the disease cannot pass between trees on opposite sides of the road because their leaves do not touch.) Etube knows that the tree at  $(-8, 2)$  is diseased. In  $(x, y)$  form, write the locations of four trees that will be infected by that tree on the first day.



- g. In Quadrant I, three trees are diseased. Write the locations  $(x, y)$  of the trees that will be infected by these three trees by the end of the first day.
- h. Etube noticed that there is one diseased tree in Quadrant III. The disease is spreading rapidly, and he is worried because the tree medicine will not arrive for four more days. How many trees in Quadrant III will still be healthy after the fourth day? How do you know? Use your diagram, keep track of the diseased trees, and **justify** your response.

1-8. Reflect on how you used the coordinates (the  $x$ - and  $y$ -values) to find and refer to points on the graph during today's lesson. Revisit the target questions, reprinted below, and share your conclusions in a class discussion.

How can you plot a point using its  $(x, y)$  coordinates?

How can you name a point on the graph?

How can you describe a pattern formed by points?

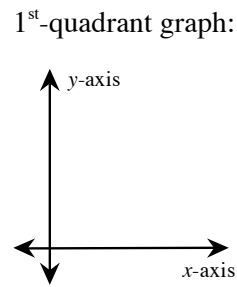
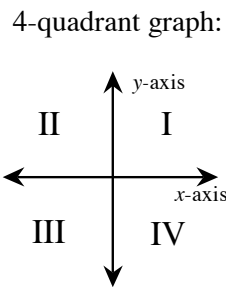


MATH NOTES

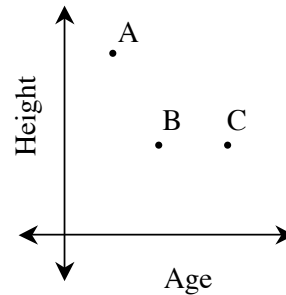
# METHODS AND MEANINGS

## Axes, Quadrants, and Coordinates

The  $x$ - and  $y$ -axes help define points on a graph (called a “Cartesian Plane”). The  $x$ -axis is horizontal, while the  $y$ -axis is vertical. The  $x$ - and  $y$ -axes divide the graphing area into four sections called **quadrants**. Written as an **ordered pair**, a point is named by its **coordinates**  $(x, y)$ , with the  $x$ -coordinate written first.



- 1-9. In the graph at right, points A, B, and C represent three different students.
- Which student is oldest? Explain how you decided.
  - What can you say about student B compared to students A and C? Tell as much as you can and **justify** your statements.



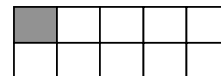
- 1-10. Compute each of the following sums and differences.
- $-7 + (-2)$
  - $3 + (-4)$
  - $-2 - 5$
  - $-1 + 5$
  - $4 - (-2)$
  - $-7 - (-8)$

- 1-11. Using whole numbers only, draw every possible rectangle with an area of 18 square units on graph paper. Write  $A = 18 \text{ un}^2$  inside each figure. Label the dimensions (length and width) of each rectangle.

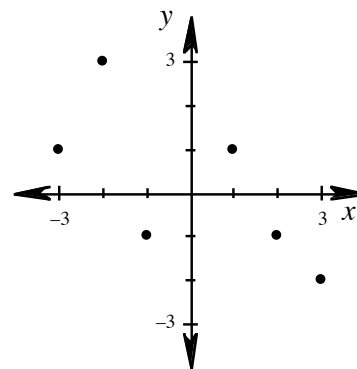
- 1-12. Latisha is determined to do well in school this year. Her goal is to maintain at least an 85% average in all of her courses.
- Latisha started with two scores: 72% and 89%. Confirm that the average of these two scores is 80.5%. Show your work.
  - Latisha's third score is 90%. Use her scores from part (a) to figure out her average now. Be sure to show your work.

- 1-13. In Algebra, you will need to be able to work with numbers, words, and geometric representations. Use these representations to answer the following problems.
- Draw and shade a figure that represents 100%. Label 100% below it. Then describe the figure in words.
  - Similarly, draw and shade figures that represent 50%, 25%, and 150%. Label each figure and describe it in words.
  - Draw and shade a figure that represents "one-third." How can this figure be represented with a number?

- d. Describe what the diagram at right represents using words and numbers.



- 1-14. Use the graph at right to answer the following questions about quadrants and coordinates of points.
- What are the coordinates of the two points in Quadrant II (where the  $x$ -value is negative and the  $y$ -value is positive)?
  - What are the coordinates of the two points in Quadrant IV (where the  $x$ -value is positive and the  $y$ -value is negative)?



### 1.1.3 How can I use data to solve a problem?



#### Collecting, Organizing, and Analyzing Data

Computing batting averages, performing scientific experiments, and polling people during elections are just a few examples of how data can provide useful information when it is collected and analyzed. In this lesson you will be collecting and organizing data to determine the potential danger of riding a roller coaster.

#### 1-15. NEWTON'S REVENGE

Have you heard about Newton's Revenge, the new roller coaster? It's so big, fast, and scary that people are already starting to talk. Some people are worried about the tunnel that thrills riders with its very low ceiling.



The closest the ceiling of the tunnel ever comes to the seat of the roller-coaster car is 200 cm. Although no accidents have been reported yet, rumors have been spreading that very tall riders have broken their arms as they went through the tunnel with their arms raised over their heads. Unfortunately, due to these rumors, many tall people have stopped riding the coaster.

**Your Task:** Consider how you could determine whether the tunnel is actually safe for any rider, no matter how tall. Discuss the questions below with your team. Be ready to share your responses with the rest of the class.

#### *Discussion Points*

What is this problem about? What is it asking you to do?

What information can help you answer this question?

How can you get the information you need?

1-16. One way to determine if the roller coaster is safe is to collect and analyze data.

- a. Collect data from each member of your team.

Each member of the team needs to be measured twice. First, have one team member stand and have another team member measure his or her height. Second, have the same student sit in a chair or desk, raise his or her arms so that they are stretched as far as possible above his or her head, and measure the distance from the seat of the chair to his or her fingertips. All measurements should be in centimeters.

Student Name	Height (cm)	Reach (cm)

Each person should record the team's data in a table like the one above.

- b. Send one person up to record your team's data on the class table. Then add the rest of the class data to your own table.
- c. Each person should put his or her initials on a sticky dot, then graph his or her own *height vs. reach* point on the class graph.

1-17. Use the class graph to answer the questions below.

- a. Are there any dots that you think show *human error*? That is, are there any dots that appear to be graphed incorrectly or that someone may have measured incorrectly? Explain why or why not.
- b. Is a person's reach related to his or her height? That is, what seems to be true about the reach of taller people? Explain.
- c. Since a person's reach depends on his or her height, we call the reach the **dependent** quantity and the height the **independent** quantity. Examine the class graph of the data from problem 1-16. On which axis was the independent data represented? On which axis was the dependent data represented?
- d. Is there a trend in the data? How can you **generalize** the trend?

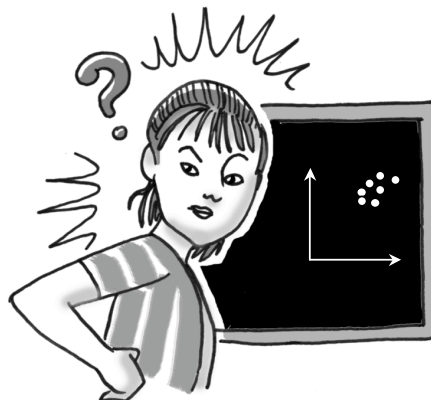
1-18. Everyone is complaining about how the teacher made the class graph!

a. Jorge is confused about how the teacher decided to set up the graph. “*Why is it a 1<sup>st</sup>-quadrant graph instead of a 4-quadrant graph?*” Answer Jorge’s question. In general, how should you decide what kind of graph to use?

b. Lauren is annoyed with the  $x$ -axis. “*Why didn’t the teacher just use the numbers from the table?*” she whined. “*Why count by twenties?*” What do you think?

c. Hosai thinks that the graph is TOO BIG. “*The dots are all mashed together! Why did the teacher begin both the  $x$ - and  $y$ -axes at zero? Anyone that short would never be allowed on the roller coaster. Why not just start closer to the smallest numbers on the table?*” she asked. What do you think?

d. Sunita says the graph is TOO SMALL! “*If we’re supposed to be using this data to check if the coaster is safe for really tall people, the graph has to have room to graph tall people’s dots too.*” Do you agree? If so, how much room do you think is needed?



1-19. Using all of your ideas from problem 1-18, make your own graph that will help you determine whether the ride is safe for very tall people. For example, the basketball player Yao Ming is 7 feet 6 inches (about 228.6 cm) tall. Is the roller coaster safe for him? Explain.

1-20. Is the roller coaster safe for all riders? Prepare a poster that shows and **justifies** your team’s answer to this question. Every team poster should include:

- A large, clear graph.
- A complete, clear, and convincing explanation of why your team thinks the ride is or is not safe for all riders.





# METHODS AND MEANINGS

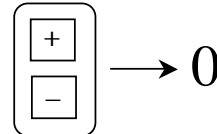
## Adding and Subtracting Integers

An **integer** is any positive or negative whole number or zero. Look at the examples of integers and non-integers below:

$-1001$     $56$     $0$     $-2$   
examples of integers

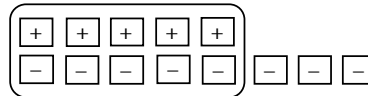
$\frac{1}{2}$     $2.1$     $\pi$     $8.3$   
examples of non-integers

The diagram at right shows that  $-1 + 1 = 0$ . One way to think of this concept is to think of an elevator. If you start one floor below ground ( $-1$ ) and travel up one floor ( $+1$ ), you end up on the ground floor (which can be represented with zero).

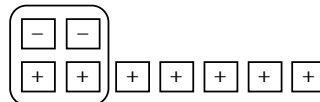


One useful strategy that works when adding and subtracting integers is to draw a diagram and eliminate zeros. Study the examples below:

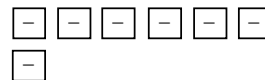
**Example 1:**  $5 - 8 = 5 + (-8) = -3$



**Example 2:**  $-2 + 7 = 5$

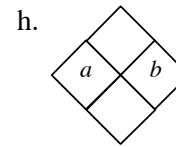
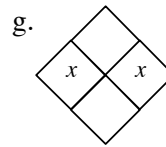
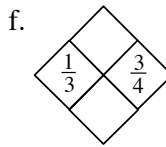
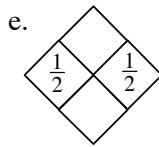
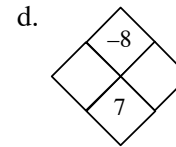
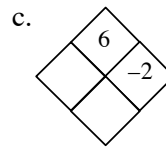
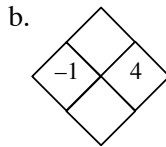
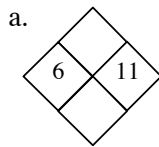
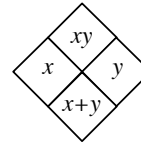


**Example 3:**  $-6 - 1 = -6 + (-1) = -7$





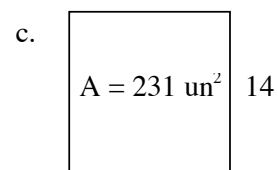
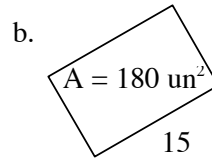
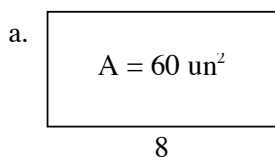
1-21. Copy these Diamond Problems and use the pattern you discovered earlier, shown at right, to complete each of them. Some of these may be challenging!



1-22. Compute without a calculator.

- |                |                |                    |                 |
|----------------|----------------|--------------------|-----------------|
| a. $-15 + 7$   | b. $8 - (-21)$ | c. $-12 - (-4)$    | d. $-9 + (-13)$ |
| e. $-50 - 30$  | f. $3 - (-9)$  | g. $-75 - (-75)$   | h. $(-3) + 6$   |
| i. $9 + (-14)$ | j. $28 - (-2)$ | k. $-3 + (-2) + 5$ | l. $3 + 2 + 5$  |

1-23. The area of each rectangle below is shown in the middle of the rectangle. For each figure, find the missing length or width.



1-24. Compute without a calculator.

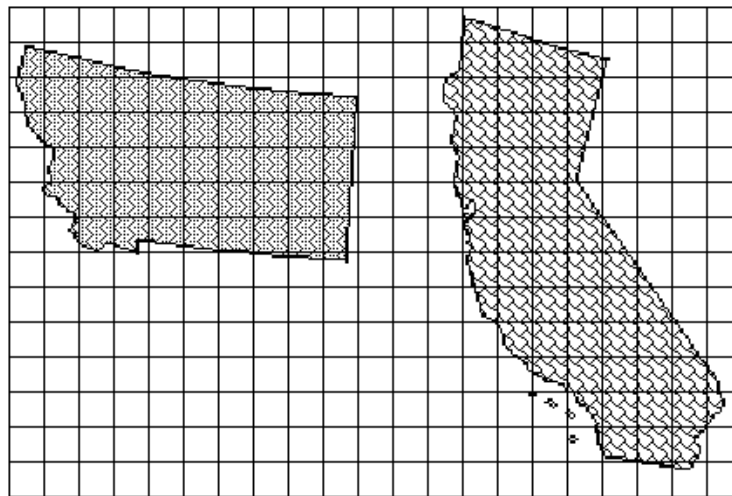
- |                 |                   |
|-----------------|-------------------|
| a. $427 - (-3)$ | b. $-50 + (-150)$ |
|-----------------|-------------------|

1-25. In Algebra, you will need to be able to work with numbers, words, and geometric representations. Use these representations to answer the following questions.

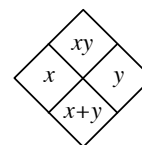
- Write another fraction that is equivalent to  $\frac{4}{5}$ . Draw diagrams to show that they are equal.
- Find the equivalent decimal for both fractions. Was rounding your answer necessary?
- Find the equivalent percent for both fractions.



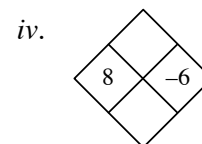
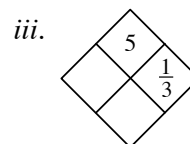
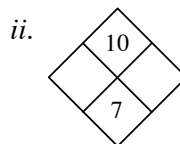
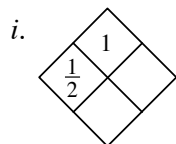
- 1-26. On graph paper, draw all of the possible rectangles with an area of 16 square units. Use only whole-number lengths (no decimals). What are the dimensions of the rectangle with the smallest perimeter?
- 1-27. The area of a rectangle is 450 square inches. If the length of the rectangle is 24 inches, what is the width?
- 1-28. Latisha earned an 85% today. Her previous scores were 72%, 89%, and 90%. Calculate her new average.
- 1-29. Estimate the areas of Montana and California using the grid below. Which state has the greatest area? Compare the area of Montana to the area of California. Explain how you estimated the area of each state.



- 1-30. The diamond at right represents the pattern you found in problem 1-4.



- a. Complete these Diamond Problems.



- b. Create two new Diamond Problems of your own.

## 1.1.4 How can I generalize a pattern?



### Finding and Generalizing Patterns

Often, mathematics is described as “the study of patterns.” Today you will preview future work with patterns by studying two tile patterns and using them to make predictions. As you work on these patterns, consider the following questions:

How do I see the pattern?

How is it changing?

Is there another way to find a solution?

#### 1-31. GROWING, GROWING, GROWING

Copy the tile pattern shown below onto graph paper.



Figure 2

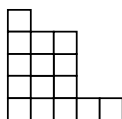


Figure 3

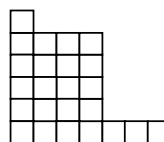


Figure 4

- Draw the 1<sup>st</sup>, 5<sup>th</sup>, and 6<sup>th</sup> figures on your paper.
- How is the pattern changing?
- What would the 100<sup>th</sup> figure look like? How many tiles would it have? How can you **justify** your prediction?

#### 1-32. Examine this new tile pattern. Copy it onto your paper.

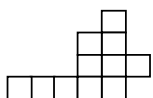


Figure 2

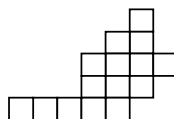


Figure 3

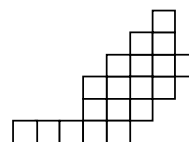



Figure 4

- Draw the 1<sup>st</sup>, 5<sup>th</sup>, and 6<sup>th</sup> figures on your paper.
- Michael knows that one of the figures in this tile pattern has 79 tiles. What is its figure number? There are many ways to figure this out – be sure to listen to each person’s ideas about how to find a solution. Be prepared to explain how you answered this question.

- 1-33. For either the pattern in problem 1-31 or the pattern in problem 1-32, prepare a team transparency or poster with your description of the pattern and your prediction. Every team transparency or poster should include:

- Clear drawings of figures from your pattern.
- An explanation of the pattern you found.
- Your prediction. (Make sure your reasoning is clear!)



MATH NOTES

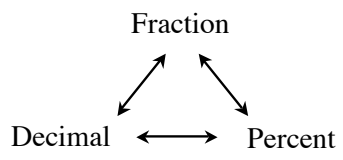
## METHODS AND MEANINGS

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### Fractions, Decimals, and Percents

Fractions, decimals, and percents are different ways to represent the same number.

Below are some ways to convert a number from one of these representations to another.



**Percent to decimal:**

Since “percent” means “out of 100,” divide the percent by 100.

$$29.6\% = 29.6 \div 100 = 0.296$$

**Fraction to percent:**

Find an equivalent fraction that has 100 in the denominator. The numerator is the percent.

$$\frac{3}{5} = \frac{60}{100} = 60\%$$

**Fraction to decimal:**

Since a fraction implies division, divide the numerator by the denominator.

$$\frac{7}{8} = 7 \div 8 = 0.875$$

**Decimal to percent:**

Reverse the process: Multiply the decimal by 100.

$$0.68 = (0.68)(100) = 68\%$$

**Percent to fraction:**

Since “percent” means “out of 100,” place the percent in a fraction over 100. Simplify as needed.

$$48\% = 0.48 = \frac{48}{100} = \frac{12}{25}$$

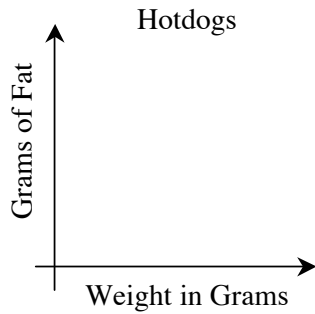
**Decimal to fraction:**

Write and reduce the fraction that has the same meaning of the decimal. For example, since 0.4 is “four-tenths”:

$$0.4 = \frac{4}{10} = \frac{2}{5}$$

**Review & Preview**

1-34. Copy the axes below onto your paper. Place and label a point on the graph for each of the products listed below.



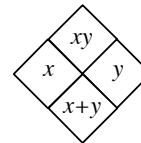
- a. Dog-Eat-Dog has a supreme hotdog that weighs 80 grams and has 40 grams of fat.
- b. Hot Doggies has a diet hotdog that weighs 50 grams and has only 9 grams of fat.
- c. Dog-alicious has a cheap hotdog that weighs 40 grams and has 30 grams of fat.



1-35. Copy and complete each sequence below. Using words, not numbers, describe how the patterns work. (For example, write, “Double the previous number.”)

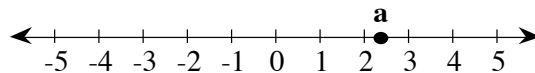
- a. 1, 3, 6, 10, \_\_\_\_, \_\_\_\_, \_\_\_\_, \_\_\_\_
- b.  $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \text{____}, \text{____}$
- c. 1, 3, 9, 27, \_\_\_\_, \_\_\_\_, \_\_\_\_, \_\_\_\_
- d. 8, 7, 5, 2, \_\_\_\_, \_\_\_\_, \_\_\_\_, \_\_\_\_
- e. 49, 47, 52, 50, 55, \_\_\_\_, \_\_\_\_, \_\_\_\_, \_\_\_\_

1-36. Recall the Diamond Problem pattern that you found in problem 1-4, which is represented in the diamond at right. Copy and complete the Diamond Problems below using the same pattern.



- a.
- b.
- c.
- d.

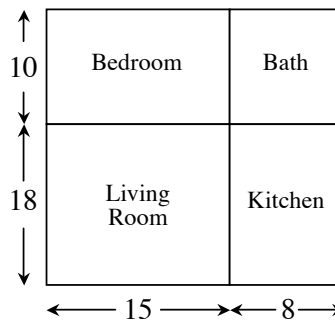
- 1-37. Copy the number line below onto your paper. Locate the following numbers by placing the lowercase letters *a* through *e* on the number line corresponding to the values given below. Part (a) is done for you.



- a.  $2\frac{1}{3}$                       b.  $-2.7$                       c.  $\frac{1}{2}$
- d.  $-0.2$                       e.  $33\frac{1}{3}\%$  of 12

- 1-38. Draw and shade a diagram to represent  $\frac{2}{3}$ . Label your diagram with the fraction. Would 0.66 or 0.67 be a more appropriate decimal equivalent? Explain why.

- 1-39. Susan's apartment is shown at right. Assuming that all rooms are rectangular, find the quantities described below. All measurements are in feet.



- a. Find the area of her living room.
- b. Find the area of her entire apartment.
- c. How much larger than her bedroom is her living room?

## 1.2.1 How can I solve it?

### Solving Problems with Guess and Check



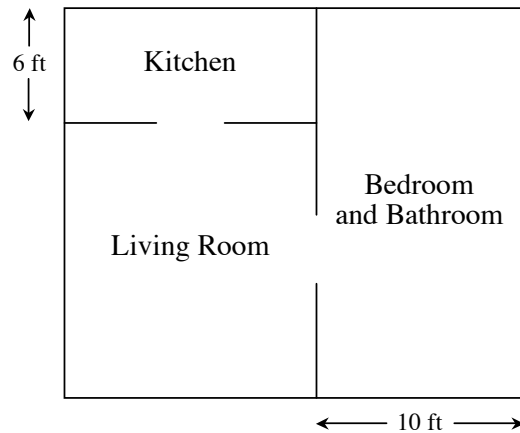
In this lesson, you will work with your team to find a strategy for solving a complex problem. It will be important for you to find ways to organize your work so that other people can follow your process.

#### 1-40. THE APARTMENT

Your architecture firm has been hired to design an apartment building. Each of the apartments in the building will be laid out as shown at right so that each room is rectangular.

The building's owners have given you the following specifications.

- The living room of an apartment must have an area of 180 square feet.
- The shorter side of the kitchen must be 6 feet to make room for counter space.
- The shorter width of the bedroom and bath must be 10 feet so that a dresser and king-sized bed will fit.
- The entire area of each apartment must be exactly 450 square feet.



**Your Task:** Find the possible dimensions for every room in the apartment. Be prepared to **justify** your answer (show how you know it works), and show all of your work in a way that someone who is not in your team can read and understand it.

### *Discussion Points*

How can you start?

How can you organize your work?

How can you use the results from one guess to make your next guess better?

## *Further Guidance*

- 1-41. Charles decided to start this problem by making a guess. He guessed that everything would come out right if one side of the living room were 10 feet.
- Does it matter which side of the living room is 10 feet long? Why or why not? Find the area of the entire apartment twice: once if the base of the living room is 10 feet long, and again if the height of the living room is 10 feet long. Do the results come out the same?
  - Is Charles' guess correct? That is, can one of the sides of the living room be 10 feet long? Explain.
  - As you checked Charles' guess, did you organize your work so that anyone could read and understand your thinking? If not, try to find a way to reorganize your work to make it clear.
  - As a team, try another guess for a dimension of the living room. Organize your work to check if that guess is correct.
- 1-42. One way to organize your work in this kind of problem is by using a table. The table can be structured something like this:

Length of _____	?	Total Area

- Use this method to check the rest of your guesses for this problem.
- Continue guessing and checking until you find correct room dimensions. Once you have an answer, organize your work so that it shows the dimensions of all of the rooms and can be read and understood by someone who is not in your team. Be ready to present to the class both your final answer and the different guesses that you tried along the way.

===== *Further Guidance* =====  
*section ends here.*


1-43. LEARNING LOGS

Throughout this course, you will be asked to reflect on your understanding of mathematical concepts in a Learning Log. Your Learning Log will contain explanations and examples to help you remember what you have learned throughout the course. It is important to write each entry of the Learning Log in your own words so that later you can use your Learning Log as a resource to refresh your memory. Your teacher will tell you where to write your Learning Log entries. Remember to label each entry with a title and a date so that it can be referred to later.



In this first Learning Log entry, describe what you know about the Guess and Check problem-solving method. For example, what does a Guess and Check problem look like? What does the solution look like? How does the method work? Title this entry “Guess and Check” and include today’s date.

MATH NOTES



## METHODS AND MEANINGS

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### Multiplication and Division of Integers

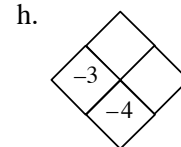
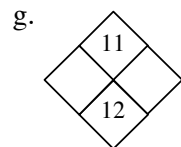
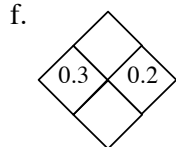
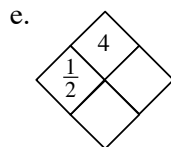
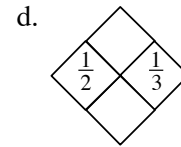
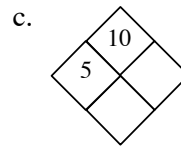
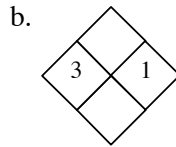
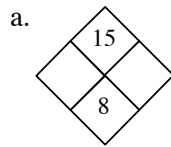
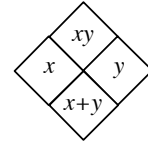
When multiplying and dividing integers, an even number of negative integers gives a positive result, and an odd number of negative integers gives a negative result. Some examples:

$(-4) \cdot (-8) = 32$	$(-60) \div (-15) = 4$	$(-5) \cdot (-2) \cdot (-4) = -40$
$(-6) \cdot (5) = -30$	$(12) \div (4) = 3$	$(-3) \cdot (5) \cdot (-4) = 60$



**Review & Preview**

1-44. Copy and complete each of the Diamond Problems below. The pattern used in the Diamond Problems is shown at right.



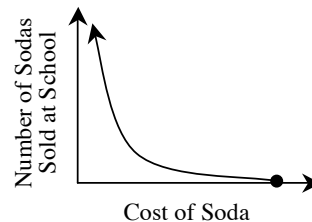
1-45. Latisha's friend Brandee forgot to make up a test and had these scores: 80%, 92%, 91%, 75%, 89%, 84%, 0%, and 85%.

- Calculate Brandee's average. Does this average score really represent her abilities? Why or why not?
- Brandee persuaded her teacher, Ms. Juarez, to allow her to make up the missed test. Brandee received a 78%. Calculate her new average.
- What difference did the 0% score make? Does this new average represent Brandee's ability more accurately?



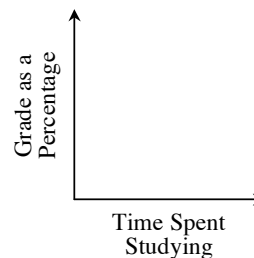
1-46. While organizing his bookshelf, Michael noticed that he had eight more science-fiction novels than spy novels. If he owns 26 science-fiction and spy novels, how many of each type of novel does he own? Explain how you found your solution.

1-47. Explain what the graph at right represents. What information does it convey?



1-48. Copy the axes at right and put a dot for each student described below.

- a. Student A, who studies hard but gets only average grades.
- b. Student B, who studies little but gets good grades.



1-49. Try these problems without a calculator first. Then use a calculator to check your answer.

- |                   |                      |                    |
|-------------------|----------------------|--------------------|
| a. $-16 + 7$      | b. $10 - (-24)$      | c. $(3)(-9)$       |
| d. $-9 + (-11)$   | e. $-49 - 36$        | f. $-56 \div (-7)$ |
| g. $15 \div (-3)$ | h. $-7 \div 7$       | i. $(-6) \cdot 9$  |
| j. $(-5)(-5)$     | k. $(-6) \cdot (43)$ | l. $27 - (-3) - 4$ |

## 1.2.2 How can I organize my work?

### More Guess and Check



As you continue your focus on Guess and Check today, organize your work so that you can find your solutions methodically.

While you work, use the following focus questions to help guide your team's discussion:

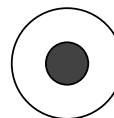
What is the problem about?

What is the relationship between the quantities involved?

How can you use the results from one guess to make a better guess?

1-50. BULL'S-EYE!

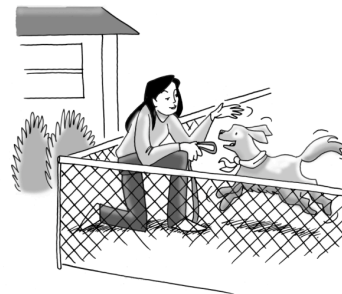
Jaime was throwing darts at a target. When his dart landed in the center of the target (the "bull's-eye"), he earned 7 points. However, when his dart landed on the outside ring, he earned 2 points. After 50 throws, his friend reported his score to be 160 points. Jaime wanted to know how many bull's-eyes he had hit, but his friend did not know. How many bull's-eyes did Jaime hit?



1-51. Your teacher will assign your team one of the following problems. Use Guess and Check to solve your problem, and then prepare a presentation to share your solution and method with the class. Use the focus questions from the lesson introduction to help you get started.

a. Adele, Amanda, and Alisa are sisters who are raising funds to go on a scouting trip. Adele sold twice as many cookies as Amanda. Alisa sold the same number as Amanda. Their uncle also donated \$15, but he did not want any cookies. Together they raised \$655. How much money did each sister raise?

b. Rachel fenced off an area in the shape of a rectangle for her dog. She used 48 feet of fencing material, and the rectangle is 6 feet longer than it is wide. What are the dimensions (length and width) of the rectangle?



c. At track practice, each runner usually carries a lap counter. Hector and McQuisten want to make their coach think that they have run farther than they actually have, so they decide to share a lap counter. Hector runs twice as far as McQuisten does. When both are finished running, the lap counter reads 48 laps. How many laps did Hector actually run?

d. When the football game started, there were twice as many Philly students as Comstock students. Five minutes into the game, busses arrived, bringing an additional 600 Philly students to the game. Total attendance for the game was 3552 students. How many students attended from each school?

e. Shaunice has 36 feet of fence to put around her rectangular flowerbed. How long and how wide will the flowerbed have to be so that she has 72 square feet of area to plant flowers?

1-52. Present to the class your team's method and solution to one of the parts of problem 1-51. As you listen to other teams' presentations, ask questions like, "*How did you know what columns to use?*", "*How did you know what to guess?*", and "*Does it matter which order the columns are in?*"

1-53. Explain in your Learning Log how you can use the results of a guess to choose your next guess. How can you tell if you should make your next guess much different than or very close to your last guess? Title this entry "Choosing Good Guesses" and include today's date.





## METHODS AND MEANINGS

### Word-Problem Vocabulary

MATH NOTES

There are some vocabulary words that occur frequently in word problems. Examine these words and their meanings below.

<b>Twice</b> • 2	This means “times two” or “double.” For example, if Erica has <i>twice</i> as many dimes as she has quarters, then if she has 6 quarters, she has $6 \cdot 2 = 12$ dimes.
<b>Sum</b> +	This is the result of adding. If a problem states that the <i>sum</i> of the number of pencils and pens is 10, that means that the number of pencils plus the number of pens is 10.
<b>Product</b> ( ) ( )	This is the result of multiplying. If a problem states that the <i>product</i> of two numbers is 96, that means that you multiply the first number by the second number to get 96.
<b>More Than</b> +	This description tells you that something is a certain quantity greater than something else. This phrase is similar to phrases like “greater than” and “higher than.” For example, if Erica has 8 <i>more</i> rabbits <i>than</i> pigs, and if she has 10 pigs, then she has $10 + 8 = 18$ rabbits. “More than” implies addition.
<b>Less Than</b> -	This description works much like “more than” (above). This phrase is similar to phrases like “fewer than” and “shorter than.” If Erica has 5 pounds <i>less</i> dog food <i>than</i> cat food, and if she has 11 pounds of cat food, then she has $11 - 5 = 6$ pounds of dog food. “Less than” implies subtraction.
<b>Times More Than</b> ( ) ( )	This description tells you that some quantity is a multiple of some other quantity. If Erica has 7 <i>times more</i> flour <i>than</i> sugar, and she has 3 pounds of sugar, then she has $7 \cdot 3 = 21$ pounds of flour. “Times more than” implies multiplication.



1-54. Solve the problem below using Guess and Check. State your solution in a sentence.

Todd is 10 years older than Jamal. The sum of their ages is 64. How old are Todd and Jamal?

1-55. Solve the problem below using Guess and Check. State your solution in a sentence.

Jabari is thinking of three numbers. The greatest number is twice as big as the least number. The middle number is three more than the least number. The sum of the three numbers is 75. Find the numbers.

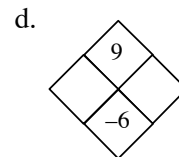
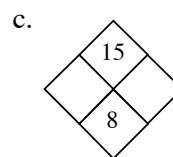
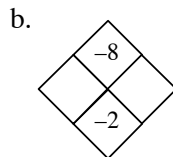
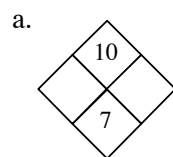
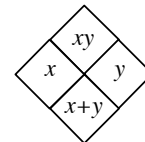
1-56. Copy the pattern at right and continue the pattern for successive powers of 3.

a. In a sentence or two, describe a pattern formed by the units digits (the “ones”) of the numbers in the pattern.

b.  $3^1 = 3$ . List the next three powers of 3 for which the ones place is a 3.

$$\begin{aligned}
 3^1 &= 3 \\
 3^2 &= 9 \\
 3^3 &= \underline{\quad} \\
 3^4 &= \underline{\quad} \\
 &\vdots \\
 &\vdots \\
 3^9 &= \underline{\quad}
 \end{aligned}$$

1-57. Copy and complete each of the Diamond Problems below. The pattern used in the Diamond Problems is shown at right.



1-58. The area of a rectangle is 24,396 square centimeters. If the width is 38 centimeters, what is the length? How do you know?

## 1.2.3 How can I solve it?

### More Guess and Check



Today you will continue your work on solving word problems with Guess and Check. As you work with your teammates, use the following focus questions to help focus your team's discussion:

What is the problem about?

What is the relationship between the quantities involved?

How can you use the results from one guess to make a better guess?

#### 1-59. HELPING THE HOMELESS

A study team decided to volunteer at the local soup kitchen to help feed the homeless people in their neighborhood. While the team cooked the soup, Rose noticed that the number of cans of tomato paste was five more than twice the number of cans of noodles. Afterward, the team recycled 44 cans that were emptied into the soup.

- How many cans of each ingredient did the team use?
- Later, Rose learned that a can of tomato paste costs \$0.70 while a can of noodles costs \$0.50. How much did it cost to make the soup?

1-60. Tamar was daydreaming during Social Studies class and only wrote down the following information: Mexico has 70 million more citizens than Canada; the United States has three times the number of citizens as Mexico; and there are 430 million citizens in Mexico, Canada, and the United States combined. Based on this information, how many citizens live in Canada?

1-61. When setting the price of admission tickets for the amusement park, Tabitha wants the price for a two-child, two-adult family to be \$100. She also wants the adult tickets to cost \$8.00 more than twice the cost of a child's ticket. How much should a child's ticket cost?



- 1-62. One side of a triangle is 3 inches shorter than twice the length of the shortest side. The third side is 5 inches longer than the shortest side. If the perimeter is 110 inches, find the lengths of all three sides.



A pet shop has 15 animals, all cats and birds. If the animals have a total of 32 legs, how many birds are there?

- 1-64. What questions do you still have about the Guess and Check solution process? In your Learning Log, write some questions you have about how to solve a word problem using Guess and Check. Title this entry “Questions about Guess and Check” and label it with today’s date.



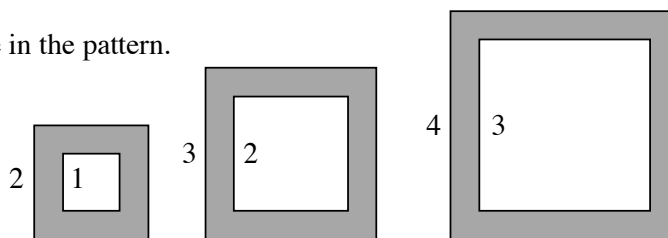
- 1-65. Solve the problem below using Guess and Check. State your solution in a sentence.

The perimeter of a triangle is 76 centimeters. The second side is twice as long as the first side. The third side is four centimeters shorter than the second side. How long is each side?

- 1-66. The pattern below is composed of nested squares.

a. Draw the next figure in the pattern.

b. Find the area of the shaded region for the figure you drew in part (a).



1-67. Examine the tile pattern at right.

- On your paper, sketch Figures 4 and 5.
- How does the pattern grow? Explain how you know.
- How many tiles will there be in Figure 100? Explain how you know.

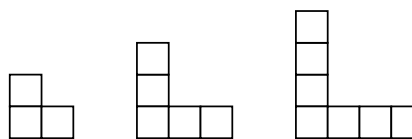
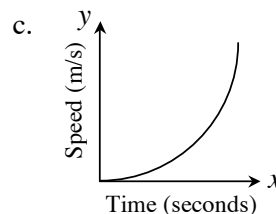
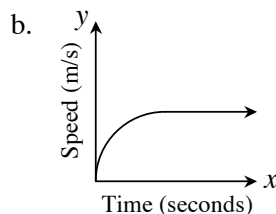
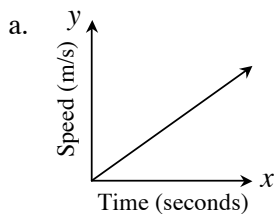
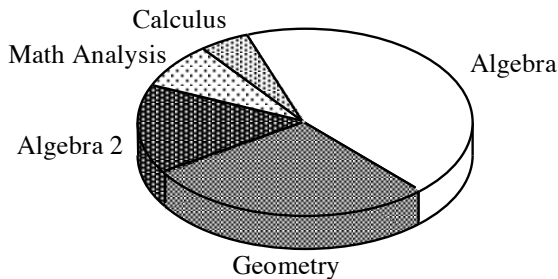


Figure 1      Figure 2      Figure 3

1-68. Look at each graph below and write a story or description about what each graph shows.



1-69. Enrollment in math courses at Kennedy High School in Bloomington, Minnesota, is shown in the pie chart at right. If there are 1000 students enrolled in math courses, approximately how many students are enrolled in Algebra? In Geometry? In Calculus?





# Chapter 1 Closure What have I learned?

## Reflection and Synthesis

The activities below offer you a chance to reflect on what you have learned during this chapter. As you work, look for concepts that you feel very comfortable with, ideas that you would like to learn more about, and topics you need more help with. Look for **connections** between ideas as well as **connections** with material you learned previously.



### ① TEAM BRAINSTORM

With your team, brainstorm a list for each of the following three topics. Be as detailed as you can. How long can you make your list? Challenge yourselves. Be prepared to share your team's ideas with the class.

**Topics:** What have you studied in this chapter? What ideas and words were important in what you learned? Remember to be as detailed as you can.

**Problem Solving:** What did you do to solve problems? What different strategies did you use?

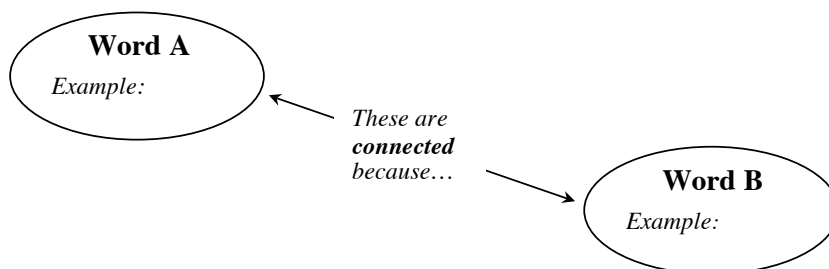
**Connections:** How are the topics, ideas, and words that you learned in previous courses are **connected** to the new ideas in this chapter? Again, make your list as long as you can.

## ② MAKING CONNECTIONS

The following is a list of the vocabulary used in this chapter. Make sure that you are familiar with all of these words and know what they mean. Refer to the glossary or index for any words that you do not yet understand.

area	average	coordinates
dimensions	equivalent	graph
Guess and Check	integers	less than
more than	pattern	perimeter
product	quadrants	sum
table	twice	$x$ - and $y$ -axes

Make a concept map showing all of the **connections** you can find among the key words and ideas listed above. To show a **connection** between two words, draw a line between them and explain the **connection**, as shown in the example below. A word can be **connected** to any other word as long as there is a **justified connection**. For each key word or idea, provide a sketch of an example.



Your teacher may provide you with vocabulary cards to help you get started. If you use the cards to plan your concept map, be sure either to re-draw your concept map on your paper or to glue the vocabulary cards to a poster with all of the **connections** explained for others to see and understand.

While you are making your map, your team may think of related words or ideas that are not listed above. Be sure to include these ideas on your concept map.

## ③ SUMMARIZING MY UNDERSTANDING

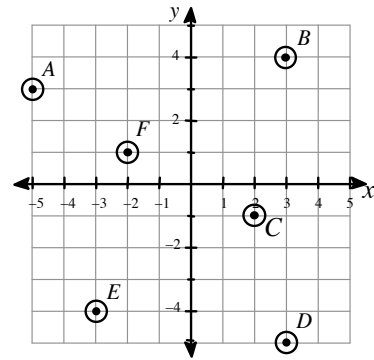
This section gives you an opportunity to show what you know about certain math topics or ideas. Your teacher will give you directions for exactly how to do this. Your teacher may give you a “GO” page to work on. “GO” stands for “Graphic Organizer,” a tool you can use to organize your thoughts and communicate your ideas clearly.

④ WHAT HAVE I LEARNED?

This section will help you evaluate which types of problems you have seen with which you feel comfortable and those with which you need more help. This section will appear at the end of every chapter to help you check your understanding. Even if your teacher does not assign this section, it is a good idea to try these problems and find out for yourself what you know and what you need to work on.

Solve each problem as completely as you can. The table at the end of this closure section has answers to these problems. It also tells you where you can find additional help and practice on problems like these.

CL 1-70. In  $(x, y)$  form, write the coordinates of each point (A through F) circled on the graph at right.



CL 1-71. On graph paper, draw and label  $x$ - and  $y$ -axes. Find and label the following points:

- $G(3, 2)$        $H(1, 4)$        $I(-2, -1)$   
 $J(4, -2)$        $K(-5, 1)$

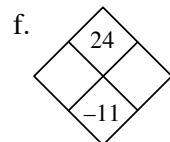
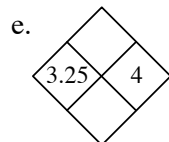
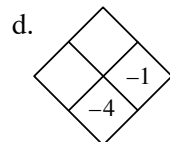
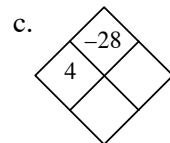
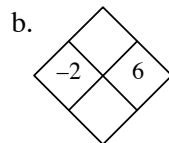
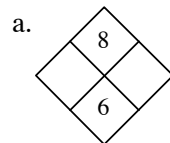
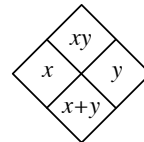
CL 1-72. Solve the following problem using Guess and Check. Show your guesses in an organized way.

Alex hangs each pair of pants and each shirt on a separate hanger. He has 51 hangers in his closet all holding clothes. Every time he buys 1 pair of pants he also buys 2 shirts. How many shirts does Alex own?



CL 1-73. On graph paper, draw at least six different-sized rectangles that have an area of 64 square units. Then find the perimeter of each rectangle.

CL 1-74. Copy and complete each of the Diamond Problems below. The pattern used in the Diamond Problems is shown at right.



CL 1-75. Copy the pattern below onto graph paper. Draw the 1<sup>st</sup> and 5<sup>th</sup> figures on your paper.



Figure 2

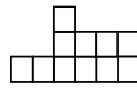


Figure 3

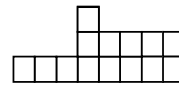


Figure 4

- How many tiles are in each figure?
- Describe how the pattern is changing.
- How many tiles would the 6<sup>th</sup> figure have? the 10<sup>th</sup> figure?

CL 1-76. Check your answers using the table at the end of this section. Which problems do you feel confident about? Which problems were hard? Have you worked on problems like these in math classes you have taken before? Use the table to make a list of topics you need help on and a list of topics you need to practice more.

### ⑤ HOW AM I THINKING?

This course focuses on five different **Ways of Thinking**: reversing thinking, justifying, generalizing, making connections, and applying and extending understanding. These are some of the ways in which you think while trying to make sense of a concept or to solve a problem (even outside of math class). During this chapter, you have probably used each Way of Thinking multiple times without even realizing it!

This closure activity will focus on one of these Ways of Thinking: **making connections**. Read the description of this Way of Thinking at right.

Think about the **connections** that you have made during this chapter. When have you linked one idea with another? What **connections** have you made with ideas that you learned in a previous math class? You may want to flip through the chapter to refresh your memory about the problems that you have worked on. Discuss any of the **connections** you have made with the rest of the class.

Once your discussion is complete, practice thinking this way by making a concept map as described in closure activity #2, *Making Connections*.

#### Making Connections

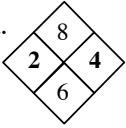
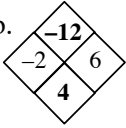
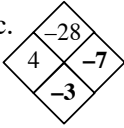
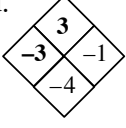
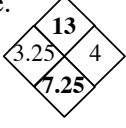
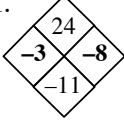
You often think this way when you try to show how one idea relates to or links with another idea. For example, when you catch yourself thinking, “*This reminds me of something...*”, you are making a connection.



## Answers and Support for Closure Activity #4

### *What Have I Learned?*

Problem	Solution	Need Help?	More Practice		
CL 1-70.	a. $(-5, 3)$ d. $(3, -5)$	b. $(3, 4)$ e. $(-3, -4)$	c. $(2, -1)$ f. $(-2, 1)$	Lesson 1.1.2 Math Notes box	Problems 1-7, 1-14, 1-34, and 1-48
CL 1-71.		Lesson 1.1.2 Math Notes box	Problems 1-7, 1-14, 1-34, and 1-48		
CL 1-72.	Alex owns 34 shirts.	Problems 1-41 and 1-42	Problems 1-50, 1-51, 1-54, 1-55, 1-59, 1-60, 1-61, 1-62, 1-63, and 1-65		
CL 1-73.	Multiple answers are possible. Rectangles with integer sides have dimensions 1 by 64, 2 by 32, 4 by 16, and 8 by 8. However, to get more than four rectangles with different dimensions, you need to also use non-integer lengths, such as $\frac{1}{2}$ by 128 or $\frac{1}{3}$ by 192.	Lesson 1.1.1 Math Notes box	Problems 1-5, 1-11, 1-23, 1-26, 1-27, and 1-58		

Problem	Solution	Need Help?	More Practice
CL 1-74.	a.  b.  c. 	Problem 1-4	Problems 1-21, 1-30, 1-36, 1-44, and 1-57
	d.  e.  f. 		

CL 1-75.



Figure 1

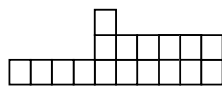


Figure 5

Lesson 1.1.4

Problems 1-31, 1-32, and 1-67

- 5, 8, 11, 14, 17
- Each figure has three more tiles than the one before it.
- The 6<sup>th</sup> figure would have 20 tiles.  
The 10<sup>th</sup> figure would have 32 tiles.