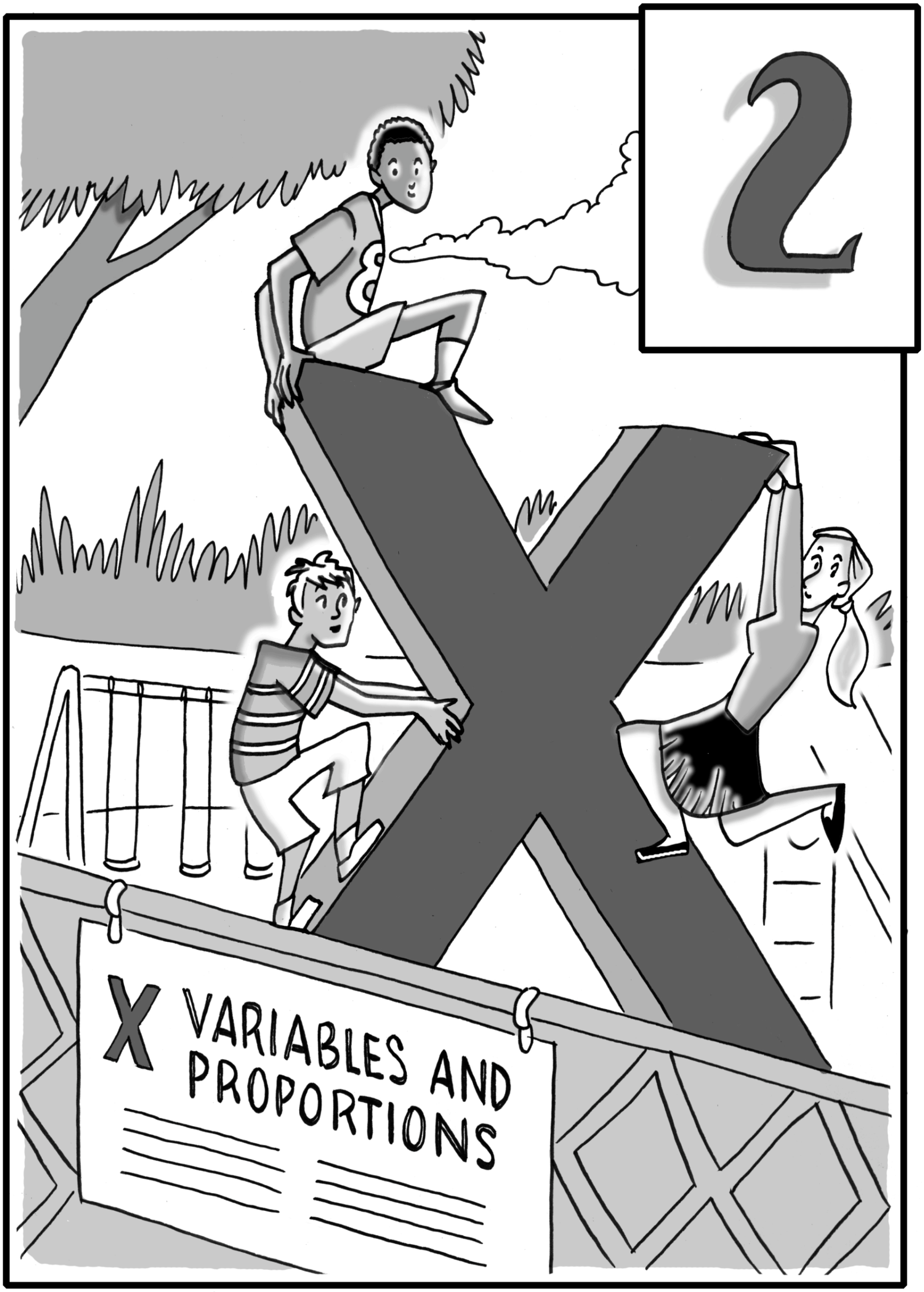

2
X VARIABLES AND PROPORTIONS

CHAPTER 2 Variables and Proportions

This chapter begins with a focus on the use of variables in algebra (such as *x* and *y*). You will use tools called "algebra tiles" to explore how and where to use variables. Since this topic lays the foundation for simplifying expressions and solving equations, it will be revisited and built upon repeatedly throughout the course.

In the second part of this chapter, you will develop methods to solve problems that involve proportional relationships. For example, if you want to know how many people at your school are left-handed, how can you use the information from your class to make a prediction? Questions like these will rely on your intuition about proportions.

Guiding Questions

Think about these questions throughout this chapter:

What is a variable?

What can I do with a variable?

How can I solve for a variable?

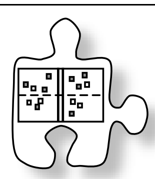
How many different ways can I write an expression?

What's the relationship?

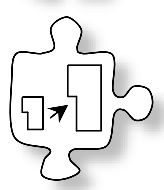
In this chapter, you will learn:

- What a variable is.
- How to write and simplify algebraic expressions.
- How to compare two complicated algebraic expressions.
- How to solve for a variable if you know that two expressions are equal.
- How to solve problems involving proportional relationships.

Chapter Outline

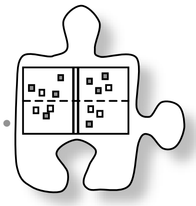


Section 2.1 This section introduces algebra tiles to develop the symbolic manipulation skills of combining like terms and solving linear equations. A special focus will be placed on the meaning of "minus" and how to make "zero."



Section 2.2 You will use your intuition about proportional relationships to find new ways to solve proportional problems.

2.1.1 What is a variable?

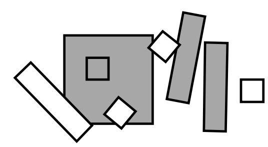


Exploring Variables and Combining Like Terms

In "The Apartment," problem 1-40, the length and width of each room was unknown. Similarly, the height of the tallest person who could ride the roller coaster safely in "Newton's Revenge," problem 1-15, was unknown. When using Guess and Check, you guessed the value of something that was unknown in order to solve for it.

In Algebra and in future mathematics courses, you will work with unknown quantities that can be represented using **variables**. Today, manipulatives called "algebra tiles" will be introduced to help you and your teammates answer some important questions, such as "What is a variable?" and "How can we use it?"

2-1. Your teacher will distribute a set of algebra tiles for your team to use during this course. As you explore the tiles, address the following questions with your team. Be prepared to share your responses with the class.



- How many different shapes are there? What are all of the different shapes?
- How are the shapes different? How are they the same?
- How are the shapes related? Which fit together and which do not?

2-2. Draw a picture of each size of tile on your paper.

a. The algebra tiles will be referred to by their areas. Since the smallest square has a length of 1 unit, its area is 1 square unit. Thus, the name for this tile is "one" or a "unit tile." Can you use the unit tile to find the other lengths? Why or why not?

b. Name the other tiles using their areas. Be sure to use what you know about the area of a rectangle and the area of a square.

2-3. JUMBLED PILES

Your teacher will show you a jumbled pile of algebra tiles and will challenge you to name all of them. What is the best description for the collection of tiles? Is your description the best possible?

2-4. Build each collection of tiles represented below. Then name the collection using a simpler algebraic expression, if possible. If it is not possible to simplify the expression, explain why not.

a. $3x + 5 + x^2 + y + 3x^2 + 2$

b. $2x^2 + 1 + xy + x^2 + 2xy + 5$

c. $2 + x^2 + 3x + y^2 + 4y + xy$

d. $3y + 2 + 2xy + 4x + y^2 + 4y + 1$

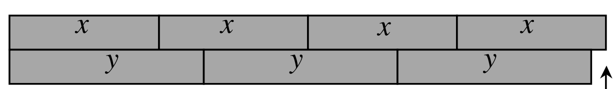
2-5. In your Learning Log, explain what a variable is in your own words. Describe each type of tile with a diagram that includes each dimension and an area label. Explain when tiles can and cannot be combined. Be sure to include examples to support your statements. Title this entry "Variables" and include today's date.

MATH NOTES

LOOKING DEEPER

Non-Commensurate

Two measurements are called **non-commensurate** if no combination of one measurement can equal a combination of the other. For example, your algebra tiles are called non-commensurate because no combination of unit squares will ever be exactly equal to a combination of x-tiles (although at times they may appear close in comparison). In the same way, in the example below, no combination of x-tiles will ever be exactly equal to a combination of y-tiles.



No matter what number of each size tile, these two piles will never exactly match.

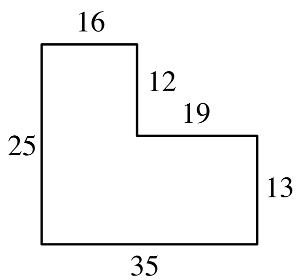
Review & Preview

2-6. Suppose you put one of your x-tiles and two unit tiles with another pile of three x-tiles and five unit tiles. What is in this new pile? Write it as a sum.

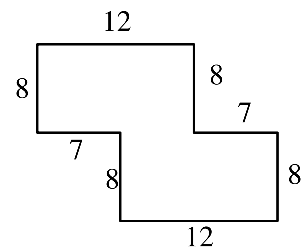
2-7. Suppose one person in your team has two x^2-tiles, three x-tiles, and one unit tile on his desk and another person has one x^2-tile, five x-tiles, and eight unit tiles on her desk. You decide to put all of the tiles together on one desk. What is the name for this new group of tiles?

2-8. Copy the following figures onto your paper. Then find the area and perimeter of each shape. Assume that all corners are right angles. Show all work.

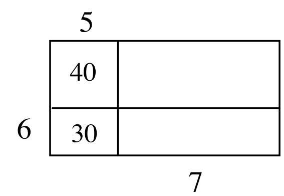
a.



b.



2-9. Find the perimeter of the entire rectangle shown at right (that is, the outside boundary of the figure). Notice that the areas of two of the parts have been labeled inside the rectangle. Also find the total area. Remember to show all work leading to your solution.



2-10. One meaning of the word **evaluate** is to find the value of an expression. To evaluate, replace a variable with a number and calculate the result. For example, when you are asked to evaluate the expression $4x-2$ when $x=-7$, you would put –7 in place of the variable and calculate: $4\cdot(-7)-2=-30$.

Evaluate the expressions below for the given values of x and y.

a. $\frac{6}{x}+9$ if $x=3$

b. $8x-3+y$ if $x=2$ and $y=1$

c. $2xy$ if $x=5$ and $y=-3$

d. $2x^2-y$ if $x=3$ and $y=8$

2-11. Use Guess and Check to solve the following problem. Write your solution as a sentence.

A cable 84 meters long is cut into two pieces so that one piece is 18 meters longer than the other. Find the length of each piece of cable.

2.1.2 What's the perimeter?

Simplifying Expressions by Combining Like Terms

While Lesson 2.1.1 focused on the area of algebra tiles, today's lesson will focus on the perimeter. What is perimeter? How can you find it? By the end of this lesson, you will be able to find the perimeter of strange shapes formed by multiple tiles.

Your class will also focus on multiple ways to find perimeter, recognizing that there are different ways to "see," or recognize, perimeter. Sometimes, with complex shapes, a convenient shortcut can help you find the perimeter more quickly. Be sure to share any insight into finding perimeter with your teammates and with the whole class.

While working today, ask yourself and your teammates these focus questions:

How did you see it?

How can you write it?

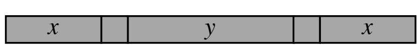
Is your expression as simplified as possible?

2-12. Your teacher will provide a set of algebra tiles for your team to use today. Separate one of each shape and review its name (area). Then find the *perimeter* of each tile. Decide with your team how to write a simplified expression that represents the perimeter of each tile. Be prepared to share the perimeters you find with the class.

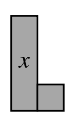
2-13. For each of the shapes formed by algebra tiles below:

- Use tiles to build the shape.
- Sketch and label the shape on your paper and write an expression that represents the perimeter.
- Simplify your perimeter expression as much as possible.

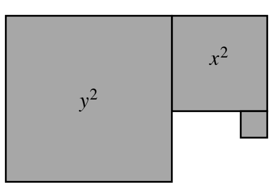
a.



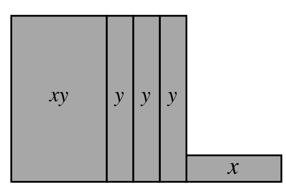
b.



c.



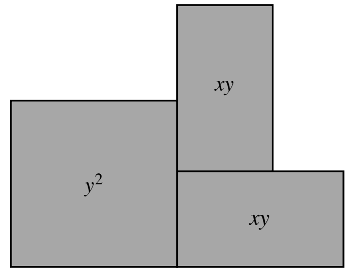
d.



2-14. Calculate the perimeter of the shapes in problem 2-13 if the length of each x-tile is 3 units and the length of each y-tile is 8 units. Show all work.

2-15. EXTENSION

The perimeter of the shape at right is 32 units. Find possible values for x and y. Is there more than one possible solution? If so, find another solution. If not, explain how you know there is only one solution.



2-16. In your Learning Log, create your own shape using three different-shaped tiles. Draw the shape and show how to write a simplified expression for its perimeter. Label this entry “Finding Perimeter and Combining Like Terms” and include today’s date.

MATH NOTES

LOOKING DEEPER

Commutative Properties

The **Commutative Property of Addition** states that when *adding* two or more number or terms together, order is not important. That is:

$a + b = b + a$ For example, $2 + 7 = 7 + 2$

The **Commutative Property of Multiplication** states that when *multiplying* two or more numbers or terms together, order is not important. That is:

$a \cdot b = b \cdot a$ For example, $3 \cdot 5 = 5 \cdot 3$

However, *subtraction* and *division* are <u>not</u> commutative, as shown below.

$$7 - 2 \neq 2 - 7 \text{ since } 5 \neq -5$$

$$50 \div 10 \neq 10 \div 50 \text{ since } 5 \neq 0.2$$

Review & Preview

2-17. Simplify each algebraic expression below, if possible. If it is not possible to simplify the expression, explain why not.

a. $3y + 2y + y^2 + 5 + y$

b. $3y^2 + 2xy + 1 + 3x + y + 2x^2$

c. $3xy + 5x + 2 + 3y + x + 4$

d. $4m + 2mn + m^2 + m + 3m^2$

2-18. Remember that one meaning of the word "evaluate" is to replace a variable with a number and to calculate the result. For example, when asked to evaluate the expression x^2 when $x = -2$, the solution would be $(-2)^2 = 4$.

Evaluate the expressions below for the given values.

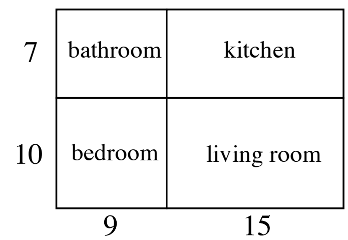
a. $-4d + 3$ if $d = -1$

b. $k - m$ if $k = 4$ and $m = -10$

c. $\frac{t}{w}$ if $t = 6$ and $w = -3$

d. $x^2 + y^2$ if $x = 7$ and $y = 5$

2-19. The diagram at right is the floor plan of Randy's apartment. All measurements are in feet. Use the diagram to answer the following questions.

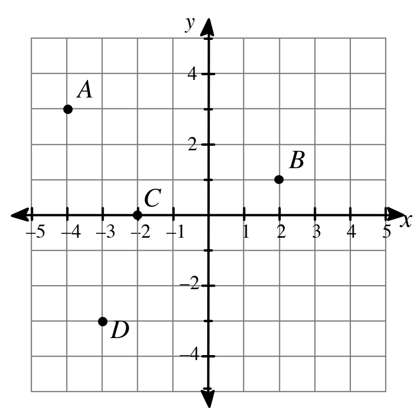


a. What are the dimensions (length and width) of Randy's living room?

b. Randy's friends are coming to visit him soon. He plans to keep them out of his bedroom. Find the area of each of the other three rooms he will have to clean.

c. What is the total area of the rooms he will have to clean?

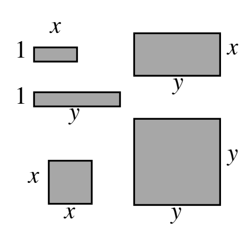
2-20. Examine the graph at right.

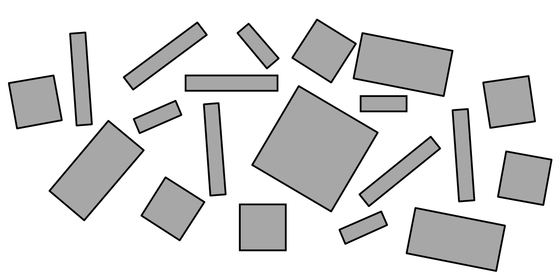


a. In (x, y) form, name the coordinates of points *A, B, C,* and *D*.

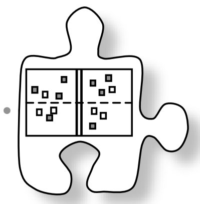
b. On graph paper, draw a set of axes like the ones shown at right. Then plot points $E(5, 2)$, $F(-3, -1)$, $G(0, -4)$, and $H(2, -3)$.

2-21. If the tiles have the dimensions shown at right, what is the name of the tile collection below? (That is, what is the total area of all of the pieces?) Write the expression algebraically, using x, x^2, y, y^2, and xy.





2.1.3 What does "minus" mean?



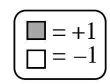
Writing Algebraic Expressions

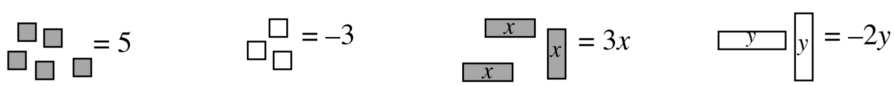
In this section, you will look at algebraic expressions and see how they can be interpreted using an expression mat. To achieve this goal, you first need to understand the different meanings of the "minus" symbol, which is found in expressions such as $5-2$, $-x$, and $-(-3)$.

2-22. What does " – " mean? Find as many ways as you can to describe this symbol and discuss how these descriptions differ from one another. Share your ideas with the class and record the different uses in your Learning Log. Title this entry "Meanings of Minus" and include today's date.

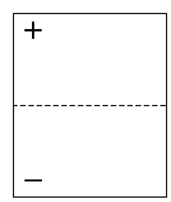
2-23. USING AN EXPRESSION MAT

Your introduction to algebra tiles in Lessons 2.1.1 and 2.1.2 involved only positive values. Today you will look at how you can use algebra tiles to represent "minus." Below are several tiles with their associated values. Note that the shaded tiles are positive and the unshaded tiles are negative (as shown in the diagram at right, which will appear throughout the text as a reminder).

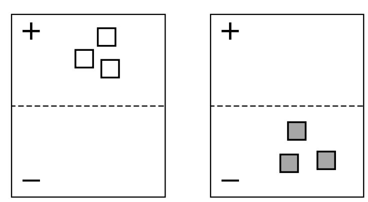




"Minus" can also be represented with a new tool called an **expression mat**, shown at right. An expression mat is an organizing tool that will be used to represent expressions. Notice that there is a positive region at the top and a negative (or "opposite") region at the bottom.



Using the expression mat, the value –3 can be shown in multiple ways, two of which are shown at right.



Value: –3 Value: –3

Note that in these examples, the left-hand diagram uses negative tiles in the "+" region, while the right-hand diagram uses positive tiles in the "–" region.

a. Build two different representations for $-2x$ using an expression mat.

b. Similarly, build $3x-(-4)$. How many different ways can you build $3x-(-4)$?

2-24. During your discussion of problem 2-23, did you see all of the different ways to represent "minus"? Discuss how you could use an expression mat to represent the different meanings discussed in class.

2-25. BUILDING EXPRESSIONS

Use the expression mat to create each of the following expressions with algebra tiles. Find at least two different representations for each expression. Sketch each representation on your paper. Be prepared to share your different representations with the class.

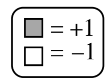
a. $-3x+4$

b. $-(y-2)$

c. $-y-3$

d. $5x-(3-2x)$

2-26. In problem 2-25, you represented algebraic expressions with algebra tiles. In this problem, you will need to **reverse** your thinking in order to write an expression from a diagram of algebra tiles.



Working with a partner, write algebraic expressions for each representation below. Start by building each problem using your algebra tiles.

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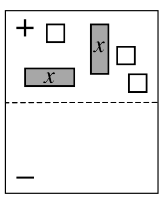

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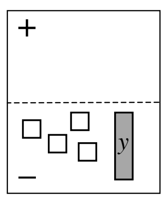
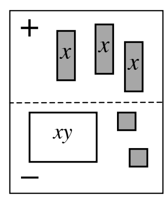
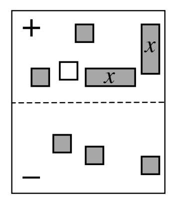


c.


d.


2-27. Patti, Emilie, and Carla are debating the answer to part (d) of problem 2-26. Patti wrote $2-1+2x-3$. Carla thinks that the answer is $2x+2-4$. Emilie is convinced that the answer is $2x-2$. Discuss with your team how each person might have arrived at her answer. Who do you think is correct? When you decide, write an explanation on your paper and **justify** your answer.

2-28. Reflect on what you have learned from today's lesson as you answer the following question in your Learning Log. Title this entry "Representing Expressions on an Expression Mat" and include today's date.

Using an expression mat, find two different ways to represent $x-1-(2x-3)$. Sketch the different representations and write a few sentences to describe the differences in the ways you built each representation.

MATH NOTES

METHODS AND MEANINGS

Evaluating Expressions and the Order of Operations

The word **evaluate** indicates that the value of an expression should be calculated when a variable is replaced by a numerical value.

For example, when you evaluate the expression $xy-4x+7$ when $x=6$ and $y=-5$, the result is:

$$(6)(-5)-4(6)+7 \Rightarrow -30-24+7 \Rightarrow -47$$

When evaluating a complex expression, you must remember to use the **order of operations**. As illustrated in the example below, the order of operations is:

First, evaluate any groups of operations that are defined by **parentheses** or other grouping symbols. $\quad 15 \div 3 \cdot 4-(8-6)^2+6$

Next, evaluate any **exponents** (such as any numbers that are squared). $\quad 15 \div 3 \cdot 4-(2)^2+6$

Then, evaluate any **multiplication** or **division** operations from left to right. $\quad 15 \div 3 \cdot 4-4+6$

$20-4+6$

Finally, evaluate any **addition** or **subtraction** operations from left to right. In this example, the expression $15 \div 3 \cdot 4-(8-6)^2+6$ has the value of 22. $\quad 22$

Review & Preview

2-29. Copy and simplify the following expressions by combining like terms. Using or drawing sketches of algebra tiles may be helpful.

a. $2x+3x+3+4x^2+10+x$

b. $4x+4y^2+y^2+9+10+x+3x$

c. $2x^2+30+3x^2+4x^2+14+x$

d. $20+5xy+4y^2+10+y^2+xy$

2-30. Read the Math Notes box for this lesson. Then evaluate each expression below.

a. For $y = 2 + 3x$ when $x = 4$, what does y equal?

b. For $a = 4 - 5c$ when $c = -\frac{1}{2}$, what does a equal?

c. For $n = 3d^2 - 1$ when $d = -5$, what does n equal?

d. For $v = -4(r - 2)$ when $r = -1$, what does v equal?

e. For $3 + k = t$ when $t = 14$, what does k equal?

2-31. Plot the points $A(5, 3)$, $B(-4, 3)$, $C(-4, -6)$, and $D(5, -6)$ on a set of axes. Use a ruler to connect them in order, including D back to A, to form a **quadrilateral** (a shape with four sides).

a. What kind of quadrilateral was formed?

b. How long is each side of the quadrilateral?

c. What is the area of the quadrilateral?

d. What is the perimeter of the quadrilateral?

2-32. Use Guess and Check to solve the following problem. Write your answer in a complete sentence.

Susan is buying three different colors of tiles for her kitchen floor. She is buying 25 more red tiles than beige tiles, and three times as many navy-blue tiles as beige tiles. If Susan buys 435 tiles altogether, how many tiles of each color does she buy?

2-33. Without a calculator, compute the value of each expression below.

a. $-14 + (-31)$

b. $-(-8) - (-2)$

c. $\frac{-16}{-8}$

d. $-11 \cdot 24$

e. $\frac{1}{2} - \frac{3}{4}$

f. $46 \div (-23)$

2.1.4 What makes zero?

Using Zero to Simplify Algebraic Expressions

Today you will continue your work with rewriting algebraic expressions. As you work with your team, ask yourself and your teammates these focus questions:

How did you see it?

How can you write it?

Is your expression as simplified as possible?

2-34. LIKELY STORY!

Imagine the following situations:

- You baby-sit your neighbor's baby and stuff the $15 you earned into your purse. When you get home, the $15 is nowhere to be found. It must have fallen out of your purse.

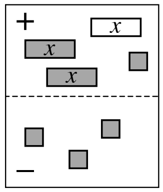
- The Burton Pumas football team completes a pass and gains 12 yards. But on the very next play, the quarterback holds onto the ball too long and gets sacked, losing 12 yards.

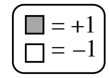
- You are at the beach. You dig a hole in the sand and place the sand you remove in a pile next to your hole. Someone comes along and pushes the pile back into the hole.

What do each of these situations have in common? Can you represent each of them using symbols? How?

2-35. How can you represent zero with tiles on an expression mat? With your team, try to find at least two different ways to do this (and more if you can). Be ready to share your ideas with the class.

2-36. Gretchen used seven algebra tiles to build the expression shown below.





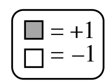
a. Build this collection of tiles in your own expression mat and write its value.

b. Represent this same value three different ways, each time using a *different number* of tiles. Be ready to share your representations with the class.

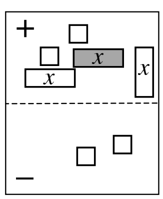
2-37. Build each expression below so that your representation does not match those of your teammates. Once your team is convinced that together you have found four different, valid representations, sketch your representation on your paper and be ready to share your answer with the class.

a. $-3x+5+y$ b. $-(-2y+1)$ c. $2x-(x-4)$

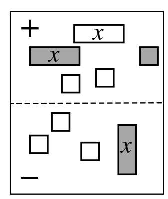
2-38. Write the algebraic expression shown on each expression mat below. Build the model and then simplify the expression by removing as many tiles as you can *without changing the value* of the expression. Finally, write the simplified algebraic expression.



a.



b.



2-39. Simplify each of the following expressions by building it on your expression mat and removing zeros. Your teacher will give you instructions about how to represent your work on your paper.

a. $3x-(2x+4)$ b. $7-(4y-3)+2y-4$

2-40. In your Learning Log, describe the different ways you can represent zero using your expression mat. Include an example and be sure to draw the tiles. Title this entry "Using Zeros to Simplify" and include today's date.

MATH NOTES

LOOKING DEEPER

Associative and Identity Properties

The **Associative Property of Addition** states that when *adding* three or more number or terms together, grouping is not important. That is:

$(a+b)+c=a+(b+c)$ For example, $(5+2)+6=5+(2+6)$

The **Associative Property of Multiplication** states that when *multiplying* three or more numbers or terms together, grouping is not important. That is:

$(a \cdot b) \cdot c=a \cdot (b \cdot c)$ For example, $(5 \cdot 2) \cdot 6=5 \cdot (2 \cdot 6)$

However, *subtraction* and *division* are not associative, as shown below.

$$(5-2)-3 \neq 5-(2-3) \text{ since } 0 \neq 6$$

$$(20 \div 4) \div 2 \neq 20 \div (4 \div 2) \text{ since } 2.5 \neq 10$$

The **Identity Property of Addition** states that adding zero to any expression gives the same expression. That is:

$a+0=a$ For example, $6+0=6$

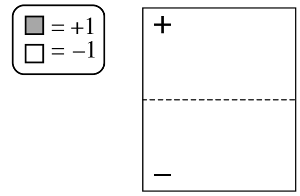
The **Identity Property of Multiplication** states that multiplying any expression by one gives the same expression. That is:

$1 \cdot a=a$ For example, $1 \cdot 6=6$

Review & Preview

2-41. Bob, Kris, Janelle, and Pat are in a study team. Bob, Kris, and Janelle have algebra tiles on their desks. Bob has two x^2-tiles, four x-tiles, and seven unit tiles; Kris has one x^2-tile and five unit tiles; and Janelle has ten x-tiles and three unit tiles. Pat's desk is empty. The team decides to put all of the tiles from the three desks onto Pat's desk. Write an algebraic expression for the new collection of tiles on Pat's desk.

2-42. Can zero be represented by any number of tiles? Using only the unit tiles (in other words, only the 1 and –1 tiles), determine if you can represent zero on an expression mat with the number of tiles below. If you can, draw an expression mat demonstrating that it is possible. If it is not possible, explain why not.



a. 2 tiles   b. 6 tiles   c. 3 tiles

2-43. Without a calculator, compute the value of each expression below.

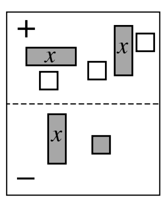
a. $-3+6$ b. $(-2)(-9)$

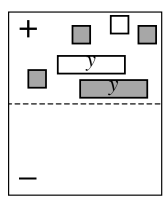
c. $-4-9-11$ d. $-12-18$

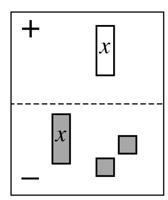
e. $\frac{3}{4}(-8)$ f. $(-2)(-2)(2)$

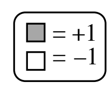
g. $7+(-19)$ h. $-15 \div 15$

2-44. Write and simplify the algebraic expression shown in each expression mat below.

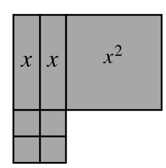
a.


b.


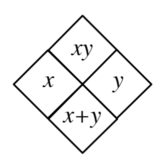
c.


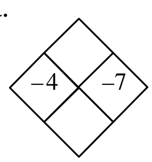


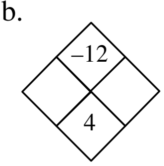
2-45. Write an expression that represents the perimeter of the shape built with algebra tiles at right.

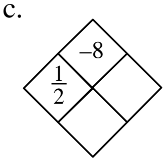


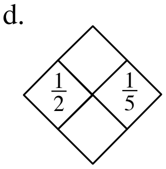
2-46. Copy and complete each of the Diamond Problems below. The pattern used in the Diamond Problems is shown at right.



a.


b.


c.


d.


2.1.5 How can I simplify the expression?

Using Algebra Tiles to Simplify Algebraic Expressions

Which is greater: 58 or 62? That question might seem easy, because the numbers are ready to be compared. However, if you are asked which is greater, $2x+8-x-3$ or $6+x+1$, the answer is not so obvious! In this lesson, you and your teammates will investigate how to compare two algebraic expressions and decide if they are equal.

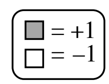
2-47. For each expression below:

- Use an expression mat to build the expression.
- Find a different way to represent the same expression using tiles.

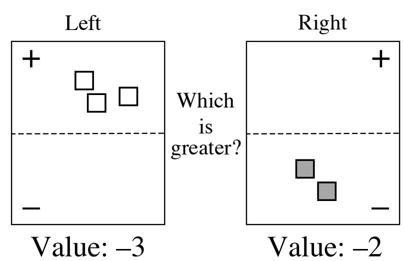
a. $7x-3$

b. $-(-2x+6)+3x$

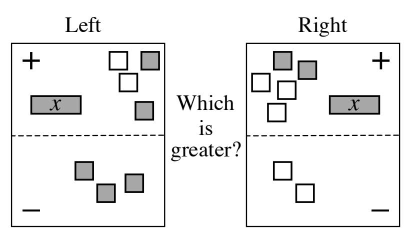
2-48. COMPARING EXPRESSIONS



Two expressions can be represented at the same time using an **expression comparison mat**. The expression comparison mat puts two expression mats side-by-side so you can compare them and see which is greater. For example, in the picture at right, the expression on the left represents –3, but the expression on the right represents –2. Since $-2 > -3$, the expression on the right is greater.



Build the expression comparison mat shown at right. Write an expression representing each side of the expression mat.

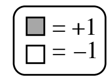


a. Can you simplify each of the expressions so that fewer tiles are used? Develop a method to simplify both sides of the expression comparison mats. Why does it work? Be prepared to **justify** your method to the class.

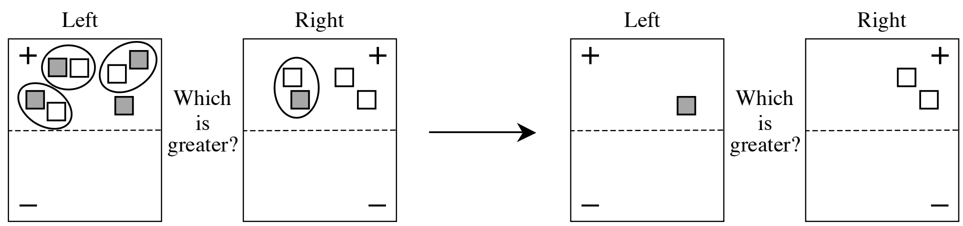
b. Which side of the expression comparison mat do you think is greater (has the largest value)? Agree on an answer as a team. Make sure each person in your team is ready to **justify** your conclusion to the class.

2-49. As Karl simplified some algebraic expressions, he recorded his work on the diagrams below.

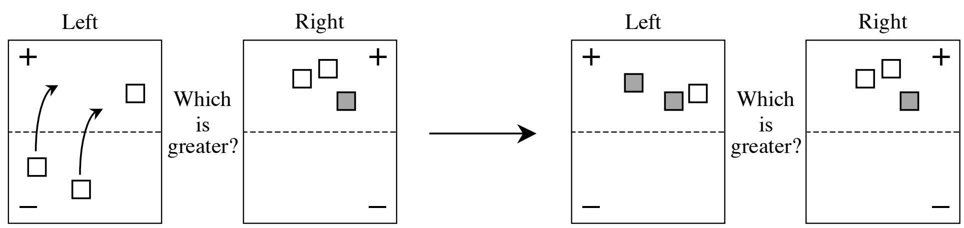
- Explain in writing what he did to each expression comparison mat on the left to get the expression comparison mat on the right.
- If necessary, simplify further to determine which expression mat is greater. How can you tell if your final answer is correct?



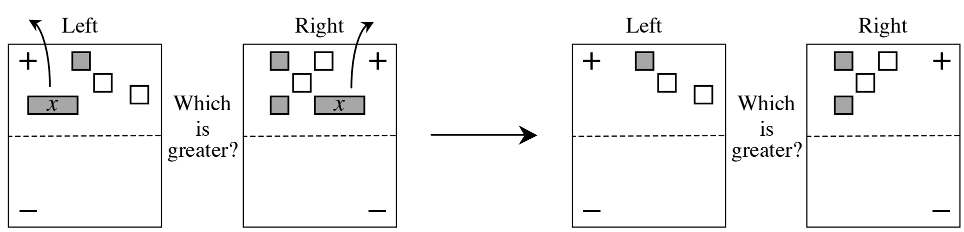
a.



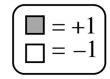
b.



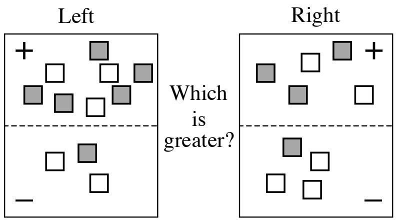
c.



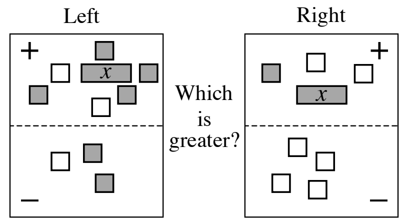
2-50. Use Karl's "legal" simplification moves to determine which side of each expression comparison mat below is greater. Record each of your "legal" moves on the Lesson 2.1.5A Resource Page by drawing on it the way Karl did in problem 2-49. After each expression is simplified, state which side is greater (has the largest value). Be prepared to share your process and reasoning with the class.



a.



b.



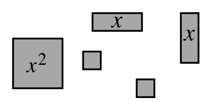
2-51. In your Learning Log, explain each of the types of "legal" moves that you can use to simplify and compare expressions. For each type of "legal" move, sketch an example. Title this entry "Legal Moves for Simplifying and Comparing Expressions" and include today's date.

MATH NOTES

METHODS AND MEANINGS

Combining Like Terms

Combining tiles that have the same area to write a simpler expression is called **combining like terms**. See the example shown at right.



$x^2 + 2x + 2$

When you are not working with actual tiles, it can help to picture the tiles in your mind. You can use these images to combine the terms that are the same. Here are two examples:

Example 1: $2x^2 + xy + y^2 + x + 3 + x^2 + 3xy + 2 \quad \Rightarrow \quad 3x^2 + 4xy + y^2 + x + 5$

Example 2: $3x^2 - 2x + 7 - 5x^2 + 3x - 2 \quad \Rightarrow \quad -2x^2 + x + 5$

A **term** is an algebraic expression that is a single number, a single variable, or the product of numerals and variables. The simplified algebraic expression in Example 2 above contains three terms. The first term is $-2x^2$, the second term is x, and the third term is 5.

2-52. Simplify the following expressions by combining like terms, if possible.

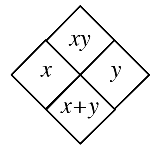
a. $x + x - 3 + 4x^2 + 2x - x$

b. $8x^2 + 3x - 13x^2 + 10x^2 - 25x - x$

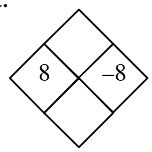
c. $4x + 3y$

d. $20 + 3xy - 3 + 4y^2 + 10 - 2y^2$

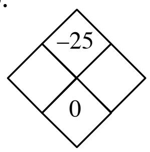
2-53. Copy and complete each of the Diamond Problems below. The pattern used in the Diamond Problems is shown at right.



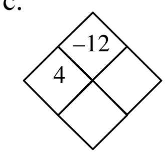
a.



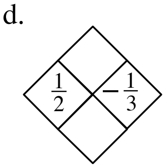
b.



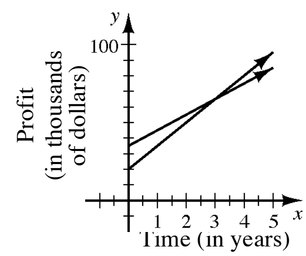
c.



d.



2-54. The two lines at right represent the growing profits of Companies A and B.



a. Sketch this graph on your paper. If Company A started out with more profit than Company B, determine which line represents A and which represents B. Label the lines appropriately.

b. In how many years will both companies have the same profit?

c. Approximately what will that profit be?

d. Which company's profits are growing more quickly? How can you tell?

2-55. Use your mental-math skills to compute the following percentages.

a. 100% of 832

b. 50% of 832

c. 25% of 832

d. 10% of 832

2-56. Evaluate each expression to find y.

a. $y = 2 + 4.3x$ when $x = -6$

b. $y = (x - 3)^2$ when $x = 9$

c. $y = x - 2$ when $x = 3.5$

d. $y = 5x - 4$ when $x = -2$

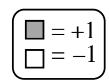
2.1.6 Which is greater?

Using Algebra Tiles to Compare Expressions

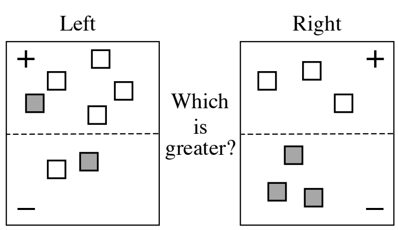
Can you always tell whether one algebraic expression is greater than another? In this lesson, you will compare the values of two expressions, practicing the different simplification strategies you have learned so far.

2-57. WHICH IS GREATER?

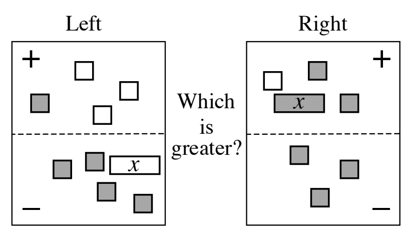
Write an algebraic expression for each side of the expression comparison mats given below. Use the "legal" simplification moves you worked with in Lesson 2.1.5 to determine which expression on the expression comparison mat is greater.



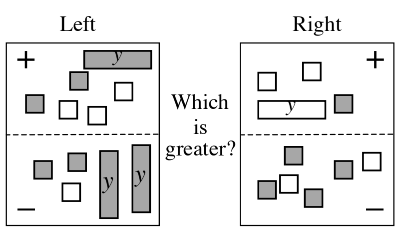
a.



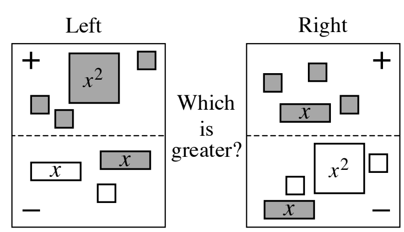
b.



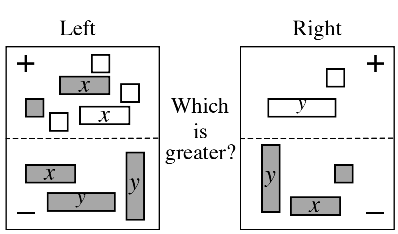
c.



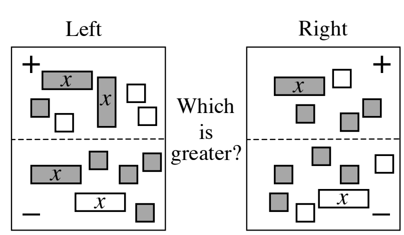
d.



e.



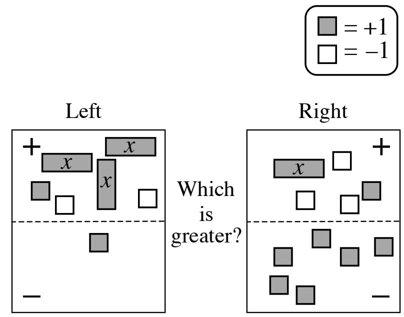
f.



2-58. Build the expression comparison mat shown below with algebra tiles.

a. Simplify the expressions using the "legal" moves that you developed in Lesson 2.1.5.

b. Can you tell which expression is greater? Explain in a few sentences on your paper. Be prepared to share your conclusion with the class.



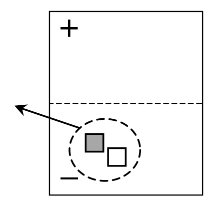
METHODS AND MEANINGS

MATH NOTES

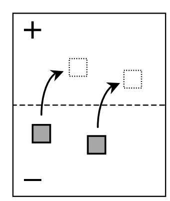
Simplifying an Expression

Three common ways to simplify or alter expressions on an expression mat are illustrated below.

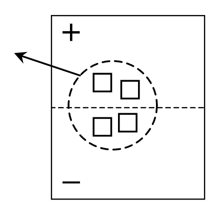
- Removing an equal number of opposite tiles that are in the same region. For example, the positive and negative tiles in the same region at right combine to make zero.



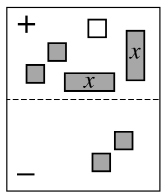
- Flipping a tile to move it out of one region into the opposite region (i.e., finding its opposite). For example, the tiles in the "–" region at right can be flipped into the "+" region.

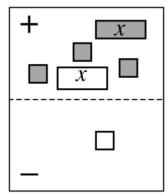


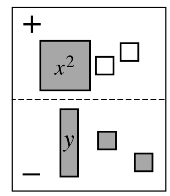
- Removing an equal number of identical tiles from both the "–" and the "+" regions. This strategy can be seen as a combination of the two methods above, since you could first flip the tiles from one region to another and then remove the opposite pairs.

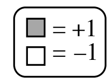


2-59. Find a simplified algebraic expression for each expression mat below.

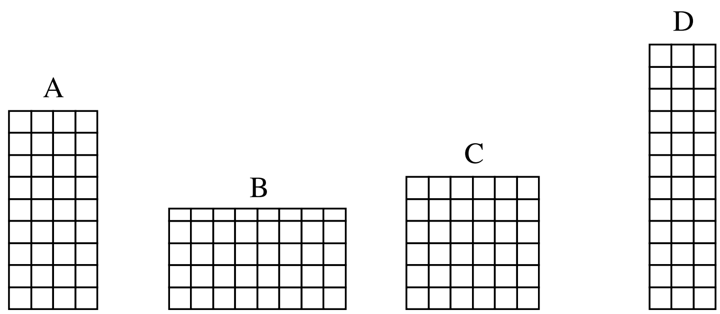
a. 

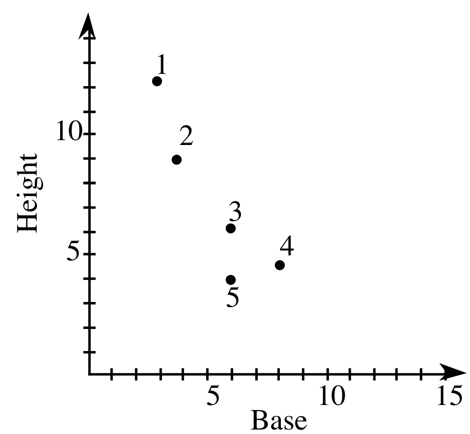
b. 

c. 



2-60. Cairo wants to create a graph that represents the heights and bases of all rectangles that have an area of 36 square units. He started by drawing the rectangles A, B, C, and D below. Examine the dimensions (length and width) of each rectangle.





a. Copy the graph at right onto graph paper. Then match the letter of each rectangle above with a point on the graph. Which point is not matched?

b. What are the base, height, and area for the unmatched point?

c. Why should the unmatched point not be on Cairo's graph?

d. Find the dimensions of three more rectangles that have areas of 36 square units. At least one of your examples should have dimensions that are not integers. Place a new point on the graph for each new rectangle you find.

e. Connect all of the points representing an area of 36 square units. Describe the resulting graph.

2-61. Without a calculator, compute the value of each expression below.

a. $7-2\cdot(-5)$

b. $6+3(7-3\cdot 2)^2$

c. $5\cdot(-3)^2$

d. $35\div(16-3^2)\cdot 2$

e. $-3\cdot 4+5\cdot(-2)$

f. $7-6(10-4\cdot 2)\div 4$

2-62. One of Teddy's jobs at home is to pump gas for his family's sedan and truck. When he fills the truck up with 12 gallons of gas, he notices that it costs him $26.28.

a. How much does one gallon of gas cost? Explain how you found your answer.

b. How much will it cost him to fill up the sedan if it needs 15 gallons of gas? Show your work.

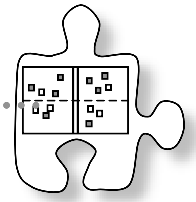
c. When Teddy filled up the tank on his moped, it cost $8.76. How much gas did his moped need? Explain how you know.

2-63. Draw a circle on your paper and lightly shade in three-fourths of the circle.

a. Divide the entire circle into eight equal parts. How many parts are shaded?

b. Using fractions, write and solve a related division problem.

2.1.7 How can I write it?



Simplifying and Recording Work

Today you will continue to compare expressions as you strengthen your simplification strategies. At the same time, you will work with your class to find ways to record your work so that another student can follow your strategies.

2-64. Use algebra tiles to build the expressions below on an expression comparison mat. Use "legal" simplification moves to determine which expression is greater, if possible. If it is not possible to tell which expression is greater, explain why.

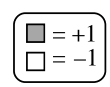
a. Which is greater: $3x-(2-x)+1$ or $-5+4x+4$?

b. Which is greater: $2x^2-2x+6-(-3x)$ or $-(3-2x^2)+5+2x$?

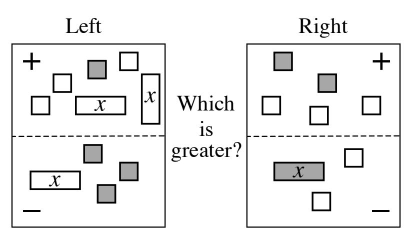
c. Which is greater: $-1+6y-2+4x-2y$ or $x+5y-(-2+y)+3x-6$?

2-65. RECORDING YOUR WORK

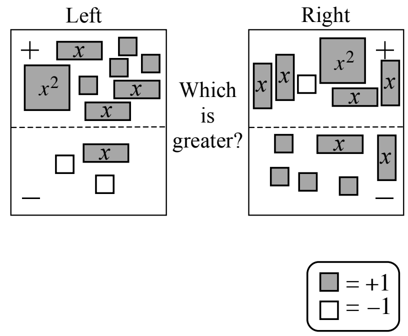
Although using algebra tiles can make some things easier because you can "see" and "touch" the math, it can be difficult to remember what you did to solve a problem unless you take good notes.



Use the simplification strategies you have learned to determine which expression on the expression comparison mat at right is greater. Record each step as instructed by your teacher. Also record the simplified expression that remains after each move. This will be a written record of how you solved this problem. Discuss with your team what the best way is to record your moves.



2-66. While Athena was comparing the expressions shown at right, she was called out of the classroom. When her teammates needed help, they looked at her paper and saw the work shown below. Unfortunately, she had forgotten to explain her simplification steps.

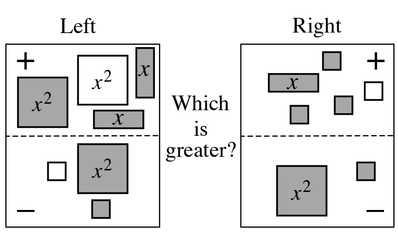


Can you help them figure out what Athena did to get each new set of expressions?

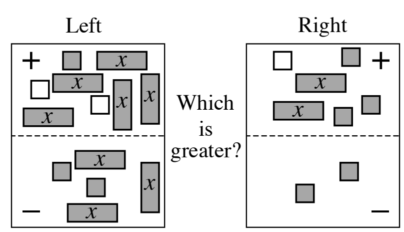
Left Expression	**Right Expression**	**Explanation**
$3x+4-x-(-2)+x^2$	$-1+x^2+4x-(4+2x)$	Original expressions
$3x+4-x-(-2)$	$-1+4x-(4+2x)$	
$3x+4-x+2$	$-1+4x-4-2x$	
$2x+6$	$2x-5$	
6	-5	
Because $6 > -5$, the left side is greater.		

2-67. For each pair of expressions below, determine which is greater, carefully recording your steps as you go. If you cannot tell which expression is greater, state, "Not enough information." Make sure that you record your result after each type of simplification. For example, if you flip all of the tiles from the "–" region to the "+" region, record the resulting expression and indicate what you did using either words or symbols. Be ready to share your work with the class.

a.



b.



c. Which is greater: $5-(2y-4)-2$ or $-y-(1+y)+4$?

d. Which is greater: $3xy+9-4x-7+x$ or $-2x+3xy-(x-2)$?

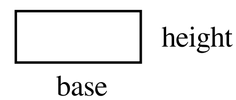
MATH NOTES

METHODS AND MEANINGS

Solving Problems with Guess and Check

By now you should have seen several ways to organize information as you solve a problem using Guess and Check. One way to organize each guess and its results is using a table. An example of this work is shown below.

Problem: The base of a rectangle is three centimeters more than twice the height. The perimeter is 60 centimeters. Find the base and height of the rectangle.



First decide what you want to guess.

Label other columns with qualities from the problem.

Height (guess)	Base ($2 \cdot$ height + 3)	Perimeter 2(height) + 2(base)	Check 60?
10	2(**10**) + 3 = 23	2(**10**) + 2(**23**) = 66	too high

Make a guess.

Use the relationships stated in the problem to determine the values of the other qualities (such as base and perimeter).

Check to see if your answer is correct. Then revise your guess and try again until you find the correct answer.

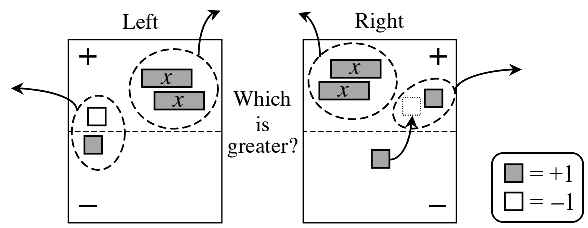
8	2(**8**) + 3 = 19	2(**8**) + 2(**19**) = 54	too low
9	2(**9**) + 3 = 21	2(**9**) + 2(**21**) = 60	correct

Review & Preview

2-68. Solve this problem using Guess and Check. You may want to review the Math Notes box for this lesson. Write your solution in a sentence.

The number of students attending the Fall play was 150 more than the number of adults attending. Student tickets cost $3, and adult tickets cost $5. A total of $4730 was collected. How many students attended the play?

2-69. Sylvia simplified the expressions on the expression comparison mat shown at right. Some of her work is shown. Are all of her moves "legal"? Explain.



2-70. Examine the tile pattern at right.

a. On graph paper, draw Figures 4 and 5.

b. What would Figure 10 look like? How many tiles would it have? What about Figure 100?

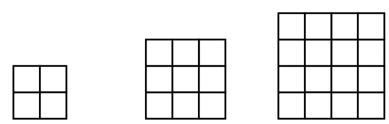
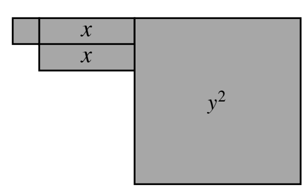


Figure 1 Figure 2 Figure 3

c. Cami has a different tile pattern. She decided to represent the number of tiles of her pattern in a table, as shown below. Can you use the table to predict how many tiles would be in Figure 5 of her tile pattern? How many tiles would Figure 8 have? Explain how you know.

Figure Number	1	2	3	4
Number of Tiles	5	9	13	17

2-71. Examine the shape made with algebra tiles at right.



a. Write an expression that represents the perimeter of the shape. Then evaluate your expression for $x = 6$ and $y = 10$ units.

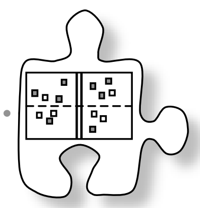
b. Write an expression that represents the area of the shape. What is the area if $x = 6$ and $y = 10$ units?

2-72. CALCULATOR CHECK

Use your scientific calculator to compute the value of each expression in the left-hand column below. Match each result to an answer in the right-hand column.

a.	$-3+16-(-5)$	1.	-16
b.	$(3-5)(6+2)$	2.	327
c.	$17(-23)+2$	3.	0.5
d.	$5-(3-17)(-2+25)$	4.	18
e.	$(-4)(-2.25)(-10)$	5.	-90
f.	$-1.5-2.25-(-4.5)$	6.	0.75
g.	$\frac{4-5}{-2}$	7.	-389

2.1.8 What if both sides are equal?



Using Algebra Tiles to Solve for x

Can you always tell whether one algebraic expression is greater than another? In this section, you will continue to practice the different simplification strategies you have learned so far to compare two expressions and see which one is greater. However, sometimes you do not have enough information about the expressions. When both sides of an equation are equal, you can learn even more about x. As you work today, focus on these questions:

How can you simplify?

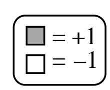
How can you get x alone?

Is there more than one way to simplify?

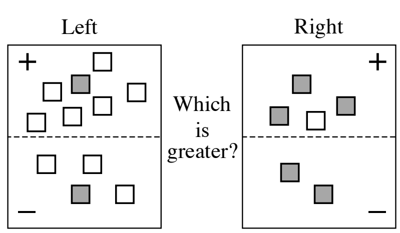
Is there always a solution?

2-73. WHICH IS GREATER?

Build each expression represented below with the tiles provided by your teacher. Use "legal" simplification moves to determine which expression is greater, if possible. If it is not possible to determine which expression is greater, explain why it is impossible. Be sure to record your work on your paper.



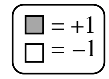
a.



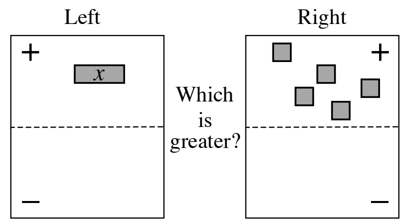
b. Which is greater: $x+1-(1-2x)$ or $3+x-1-(x-4)$?

2-74. WHAT IF BOTH SIDES ARE EQUAL?

If the number 5 is compared to the number 7, then it is clear that 7 is greater. However, what if you compare x with 7? In this case, x could be smaller, larger, *or equal to* 7.



Examine the expression comparison mat below.



a. If the left expression is smaller than the right expression, what does that tell you about the value of x?

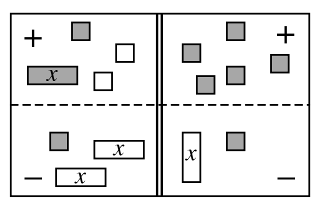
b. If the left expression is greater than the right expression, what does that tell you about the value of x?

c. What if the left expression is equal to the right expression? What does x have to be for the two expressions to be equal?

2-75. SOLVING FOR *X*

Later in the course, you will learn more about situations like parts (a) and (b) in the preceding problem, called "inequalities." For now, assume that the left expression and the right expression are equal in order to learn more about x. The two expressions will be brought together on one mat to create an **equation mat**, as shown in the figure below. The double line down the center of an equation mat represents the word "equals." It is a wall that separates the left side of an equation from the right side.

a. Obtain the "Equation Mat" resource page from your teacher. Build the equation represented by the equation mat at right using algebra tiles. Simplify as much as possible and then solve for x. Be sure to record your work.



b. Build the equation $2x-5=-1+5x+2$ using your tiles by placing $2x-5$ on the left side and $-1+5x+2$ on the right side. Then use your simplification skills to simplify this equation as much as possible so that x is alone on one side of the equation. Use the fact that both sides are equal to solve for x. Record your work.

2-76. Now **apply** this new solving skill by building, simplifying, and solving each equation below for x. Record your work.

a. $3x-7=2$

b. $1+2x-x=x-5+x$

c. $3-2x=2x-5$

d. $3+2x-(x+1)=3x-6$

e. $-(x+3-x)=2x-7$

f. $-4+2x+2=x+1+x$

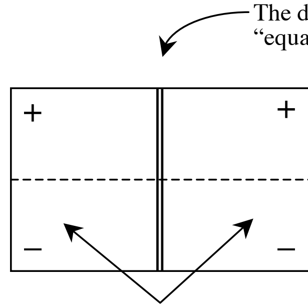
METHODS AND MEANINGS

MATH NOTES

Using an Equation Mat

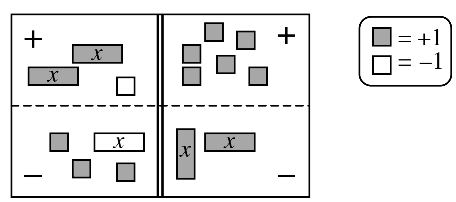
An **equation mat** can help you visually represent an equation with algebra tiles.

The double line represents the "equals" sign (=).



For each side of the equation, there is a positive and a negative region.

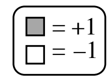
For example, the equation $2x-1-(-x+3)=6-2x$ can be represented by the equation mat at right. (Note that there are other possible ways to represent this equation correctly on the equation mat.)



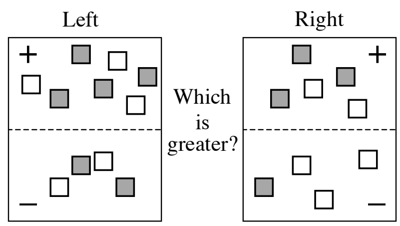
Review & Preview

2-77. WHICH IS GREATER?

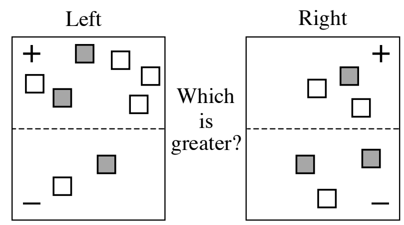
For each expression comparison mat below, simplify and determine which side is greater.



a.



b.



2-78. Use Guess and Check to solve the problem below. Then state your solution in a sentence.

Mairé is thinking of two numbers. When she adds them, she gets 40. When she multiplies them, she gets 351. Help her younger sister, Enya, figure out the numbers.

2-79. Simplify each expression below as much as possible.

a. $3y - y + 5x + 3 - 7x$

b. $-1 - (-5x) - 2x + 2x^2 + 7$

c. $6x + 2 - 1 - 4x - 3 - 2x + 2$

d. $\frac{2}{3}x - 3y + \frac{1}{3}x + 2y$

2-80. Plot the points (0, 0), (3, 2), and (6, 4) on graph paper. Then draw a line through the points. Name the coordinates of three more points on the same line.

2-81. Mr. Dexter's teams earned the following scores on a quiz: 15, 20, 19, 20, 16, 20, 14, 18, and 17.

a. What is the mean (average score)?

b. What is the median (middle score)?

c. What is the mode (the score that occurs most often)?

2.1.9 What is x?

More Solving Equations

Today you will explore more equations on the equation mat and will examine all of the tools you have developed so far to solve for x. While you are working on these problems, be prepared to answer the following questions:

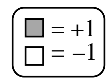
How can you simplify?

Can you get the variable alone?

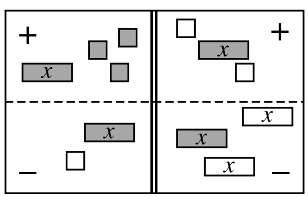
Is there more than one way to simplify?

Is there always a solution?

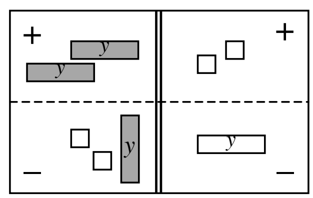
2-82. On your paper, write the equation represented in each diagram below. For each equation, simplify as much as possible and then solve for x or y. Be sure to record your work on your paper.



a.

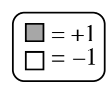


b.

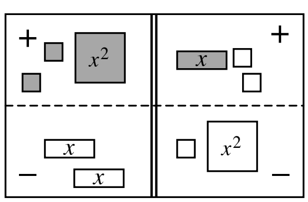


2-83. IS THERE A SOLUTION?

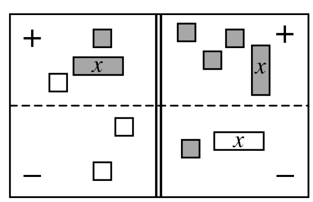
While solving homework last night, Richie came across three homework questions that he thinks have no solution. Build each equation below and determine if it has a solution for x. If it has a solution, find it. If it does not have a solution, explain why not.



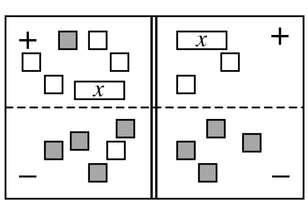
a.



b.



c.



2-84. Continue to develop your equation-solving strategies by solving each equation below (if possible). Remember to build each equation, simplify as much as possible, and solve for *x* or *y*. There are often multiple ways to solve equations, so remember to **justify** that each step is "legal." If you cannot solve for *x*, explain why not. Be sure to record your work.

a. $-x+2=4$

b. $4x-2+x=2x+8+3x$

c. $4y-9+y=6$

d. $9-(2-3y)=6+2y-(5+y)$

2-85. In your Learning Log, explain when you can solve for *x* in an equation and when you cannot. Be sure to give an example of each situation. Title this entry "Solutions of an Equation" and include today's date.

MATH NOTES

LOOKING DEEPER

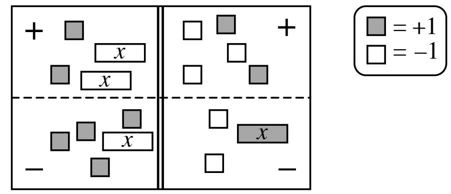
Inverse Properties

The **Additive Inverse Property** states that for every number a there is a number $-a$ such that $a+(-a)=0$. A common name used for the additive inverse is the **opposite**. That is, $-a$ is the opposite of a. For example, $3+(-3)=0$ and $-5+5=0$.

The **Multiplicative Inverse Property** states that for every nonzero number a there is a number $\frac{1}{a}$ such that $a \cdot \frac{1}{a}=1$. A common name used for the multiplicative inverse is the **reciprocal**. That is, $\frac{1}{a}$ is the reciprocal of a. For example, $6 \cdot \frac{1}{6}=1$.

Review & Preview

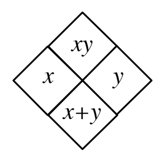
2-86. Translate the equation mat at right into an equation. Remember that the double line represents "equals."

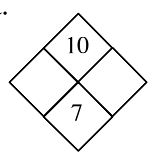


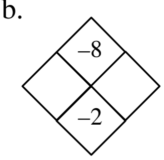
2-87. Ling wants to save \$87 for tickets to a rock concert. If she has \$23 now and will save \$4 per week, how long will it take her to get enough money to buy the tickets? Make a Guess and Check table to help you solve this problem.

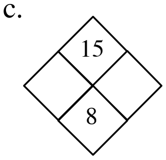
2-88. On graph paper, plot the points (0, 0), (–2, 1), and (2, –1). Then draw a line through them. Name the coordinates of three more points on the same line that have integer coordinates.

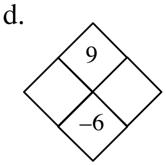
2-89. Copy and complete each of the Diamond Problems below. The pattern used in the Diamond Problems is shown at right.



a.


b.


c.


d.


2-90. Evaluate the expressions below for the given values.

a. $6m + 2n^2$ for $m = 7$ and $n = 3$

b. $\frac{5x}{3} - 2$ for $x = -18$

c. $(6x)^2 - \frac{x}{5}$ for $x = 10$

d. $(k-3)(k+2)$ for $k = 1$

2.2.1 How can I solve it?

Solving Problems With Proportional Intuition

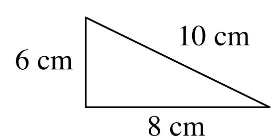
In Chapter 1, you looked at ways to organize your algebraic thinking using graphs and tables. In the first part of this chapter, you used algebra tiles to model combining like terms and solving equations. Today you will examine proportional situations. What proportional tools do you already have? What new ways can you and your team find to solve proportional problems?

2-91. PROPORTIONAL RELATIONSHIPS

Solve the five problems (a) through (e) below. Don't estimate! Instead, use the information to find an answer that is as accurate as possible. There are many different strategies you can use, so be sure to give reasons as you discuss your ideas with your team. To help **justify** your ideas, make sure both to *show* and to *explain* your work. For example:

- Label all numbers with what they represent. For example, don't just write "5"; instead, write "5 pounds."
- Give reasons for each action. For example, if you decided to add 10, say why. (Why did you add? Why 10?)
- Organize your work so that others will understand what you did and why you did it that way.

a. Mr. Douglas made a copy of the triangle shown at right, but he accidentally enlarged it! The longest side of the new copy had a length of 32 cm. How long are the two shorter legs?



b. Ferroza's pet ferret eats so much that she has to buy ferret food in bulk. Five pounds cost $17.50. How much would 30 pounds cost? How much would 33 pounds cost?

Problem continues on next page →

2-91. *Problem continued from previous page.*

c. Oscar often cleans his teachers' overhead transparencies. He can clean 17 transparencies in 10 minutes. At this rate, how long would it take him to clean 75 transparencies? Now **reverse** the problem: How many transparencies could Oscar clean in one hour?

d. 

The Math Club is having a tamale sale! The school has 1600 students, but the club members are not sure how many tamales to make. One day during lunch, the club asked random students if they would buy a tamale. They found that 15 out of 80 students surveyed said they would definitely buy a tamale. How many tamales should the Math Club expect to sell?

e. When he was little, Miguel could not sleep without his Captain Terrific action figure – it looked so life-like because it was a perfect scale model. The actor who plays Captain Terrific on television is 216 cm tall. Miguel's doll is 10 cm tall.

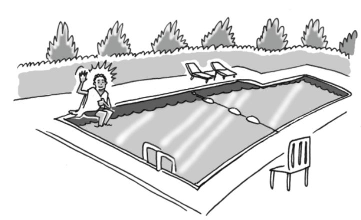
i. If the doll's neck is 0.93 cm long, how long is the actor's neck?

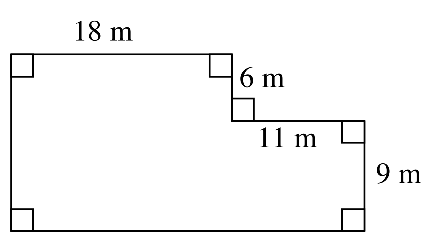
ii. If the actor's head has a circumference of 30 cm, what is the circumference of the doll's head?

2-92. The five problems above are all examples of situations that involve **proportional relationships**. In your Learning Log, write your observations about what all five of these problems have in common. Title this entry "Proportional Relationships" and label it with today's date.

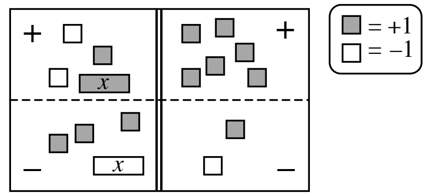
Review & Preview

2-93. Find the perimeter and area of Jacob's swimming pool shown in the diagram below. Be sure to show all of your work.





2-94. On your paper, write the equation represented in the equation mat at right. Simplify as much as possible and then solve for x.



2-95. For each equation below, draw a picture of the tiles in an equation mat, simplify, and solve for x. Record your work.

a. $2x - 7 = -x + 2$

b. $-2 - 3x = x + 6$

2-96. Copy and simplify the following expressions by combining like terms.

a. $y + 2x - 3 + 4x^2 + 3x - 5y$

b. $2x - 6x^2 + 9 - 1 - x - 3x$

c. $2y^2 + 30x - 5y^2 + 4x - 4y - y$

d. $-10 + 3xy - 3xy + y^2 + 10 - y^2$

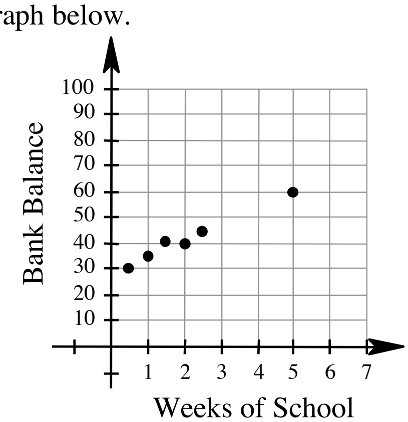
2-97. Ferroza can buy a 24-ounce bag of ferret food for \$1.19, or she can buy a 36-ounce bag for \$2.89. Which is the better deal? **Justify** your conclusion.

2-98. Since the beginning of school, Steven has been saving money to buy a new MP3 player. His bank balance is represented by the graph below.

a. According to the graph, about how much money had Steven saved after 2 weeks of school?

b. About how much money did Steven probably have after 4 weeks of school? How can you tell?

c. If he keeps saving at the same rate, how much will he have saved by Week 7? Explain how you know.



2.2.2 How can I organize my work?

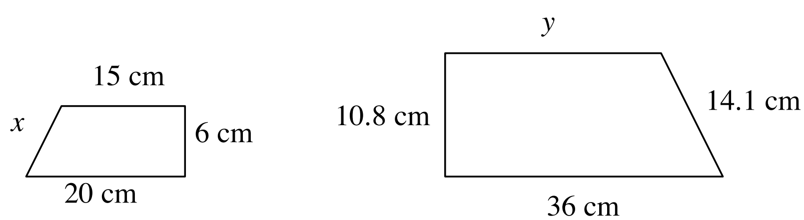
Sharing Proportion-Organizing Strategies

In Lesson 2.2.1, you solved problems using proportional reasoning (that is, you used the fact that each situation was proportional to solve the problem). Today you will work with two more proportional situations. This time, your focus will be on how to organize your work and how to explain your reasoning.

2-99. Solve these proportion problems and be ready to present your method. Remember to label all numbers and explain (in words or symbols) the reasons for your work. Be sure to organize your ideas in a way that will help others see what you did.

a. Toby uses seven tubes of toothpaste every ten months. How many tubes would he use in five years? In two years? How long would it take him to use 100 tubes?

b. Mr. Douglas is at it again! The little trapezoid below on the left got enlarged in the photocopying machine and turned into the big trapezoid on the right. How long are the missing sides of the shapes (x and y)?



2-100. Look back at the proportion problems you solved today and yesterday. Share your strategies with the class.

a. Did you always organize your work the same way? What different organizational strategies did you or your classmates use?

b. What did all of the problems yesterday and today have in common?

2-101. In your Learning Log, show at least two examples of ways to organize your work when solving problems that require proportional reasoning. Title this entry "Solving Proportion Problems" and include today's date.

MATH NOTES

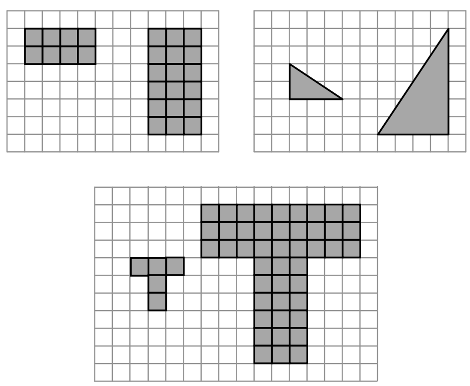
METHODS AND MEANINGS

Similar Figures

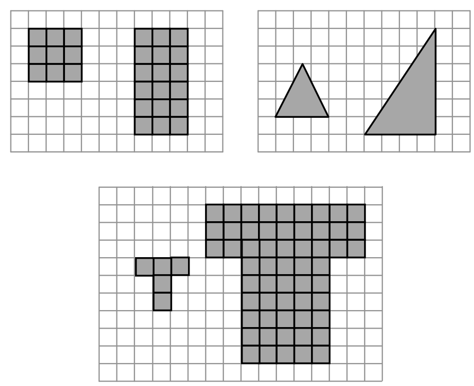
Photocopy-enlargement problems like Mr. Douglas's (see part (a) of problem 2-91 and part (b) of problem 2-99) involve figures that are similar. In plain English, "similar" means "close to the same." But in mathematics, **similar** means that two figures have the same shape but are not necessarily the same size.

Below are examples of some shapes that are similar and other shapes that are not similar.

These pairs of shapes <u>are</u> similar:



These pairs are <u>not</u> similar:



Review & Preview

2-102. Use proportional reasoning to solve the following problems.

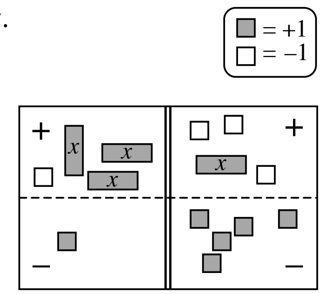
a. A typical small bag of colored candies has about 135 candies in it, 27 of which are blue. At this rate, how many blue candies would you expect in a pile of 1000 colored candies?

b. Ten calculators cost \$149.50. How much would 100 cost? 1000? 500?

c. Tickets to 50 home baseball games would cost \$1137.50. How much would it cost to get tickets for all 81 home games? How many games could you go to for \$728?

2-103. At the annual dog show, Chantel noticed that there were three more Schnauzers than Scotties. She also realized that the number of Wirehaired Terriers was three less than twice the number of Schnauzers. If there were 78 dogs in all (counting Schnauzers, Scotties, and Wirehaired Terriers), how many Schnauzers were there?

2-104. Write the equation represented in the equation mat below.



a. Simplify both sides as much as possible, and then solve for x.

b. Evaluate both the left side and the right side using your solution from part (a). Remember that if your solution is correct, both sides should have the same value.

2-105. For the following equations, draw a picture of the tiles in an equation mat, use "legal" moves to simplify, and then solve for x. Record your work.

a. $-3x+7=x-1$

b. $1+2x-3=4x-2$

2-106. To bake 100 of his favorite cookies, Mr. Wallis needs 350 grams of sugar.

a. How many grams of sugar would he need to bake 10 cookies? What about 20 cookies? Show all work.

b. To help him know how much sugar to use when baking cookies, Mr. Wallis started to make a table, as shown at right. Copy and complete his table on your paper.

Cookies	**Sugar (g)**
100	350
10	
20	
2	
1	
5	
	700
	1400

2-107. Read the Math Notes box for this lesson, entitled "Similar Figures." Then draw your own example of two figures that are **similar**.

2.2.3 How can I use proportional relationships?

Using Proportional Relationships

How many books in the Library of Congress are science fiction? How many people will vote for your favorite candidate in the next election? How many fish are in the ocean?

To answer questions about huge quantities, it is sometimes best to start by thinking in terms of small quantities. Today you will use proportional relationships to make predictions about large collections of people and things. During today's work, consider:

How are these related?

How can you use this information to make a prediction?

2-108. COUNTING CANDIES

Can you use the number of candies in a small bag to predict the number of candies in a large bag? To answer this, follow the directions from your teacher and discuss these questions:

- Using the small bag of candy, what could you predict about the large bag?
- Could you also use the large bag of candy to make a prediction about the small bag? Why or why not?

a. Decide what you will predict about the large bag. For example, you could try to predict the number of red candies in the large bag.

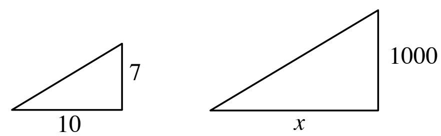
b. Collect data from the small bag. What should you measure?

c. Use your data and proportional reasoning to make a prediction about the large bag.

d. As a class, find a way to determine if your prediction was fairly accurate. If it was not close to being accurate, what do you think happened?

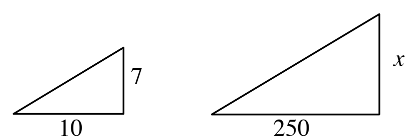
2-109. Kenny can make seven origami (folded-paper) cranes in ten minutes. He read a story about a girl who made 1000 cranes, and was curious about how long it would take him to make that many (assuming he worked without stopping).

a. Instead of solving this problem using tables, Kenny represented it using similar triangles. He drew the diagram below. What do the corresponding parts of his two triangles represent?



b. Use the diagram in part (a) to finish Kenny's geometric solution. How long would it take him to make 1000 cranes?

c. Next, Kenny drew the two similar triangles at right for a related problem. What question do you think he is trying to answer? In other words, what do you think x would represent? Find x.



2-110. How many people are in your math class? How many of those people are left-handed? Use this information to make predictions about other groups of people.

a. How many students are in your grade? About how many of those students would you expect to be left-handed?

b. How many students are in your entire school? About how many of those students would you expect to be left-handed?

c. Now **reverse** the process: The Kennedy Middle School Left-Handers' Club has counted 270 left-handed students in their school. About how many students do you think there are at Kennedy Middle School?

d. Across town, Grand Prairie High has 1060 *right*-handed students. About how many students would you expect there to be at Grand Prairie High?

2-111. Where else does the idea of a proportional relationship between part and whole appear? Read the Math Notes box following this problem and explain how one part of the Sierpinski Triangle is similar to the whole triangle.

MATH NOTES

LOOKING DEEPER

Fractals

Fractals are geometric structures developed by repeating a process over and over. A famous example of a fractal is the **Sierpinski Triangle**, shown below. To create this design, make three copies of a triangle and place them into a larger triangle, as shown in Figure 1. Then repeat the process by taking the large triangle of Figure 1 and copying it in the same arrangement, shown in Figure 2. If this process is continued infinitely, the result is the Sierpinski Triangle.

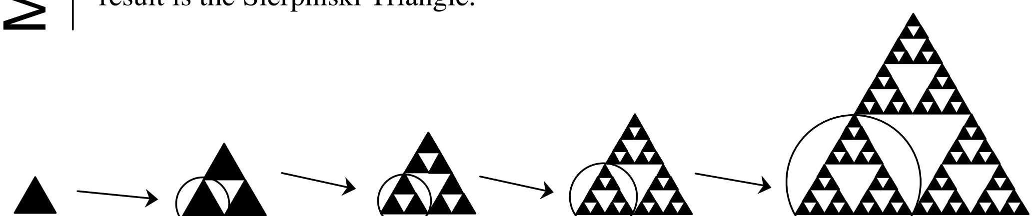
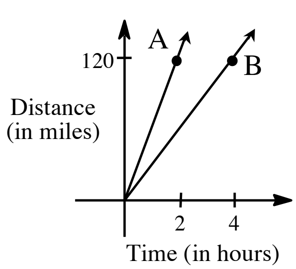


Figure 0 Figure 1 Figure 2 Figure 3 Figure 4

Review & Preview

2-112. When baking cookies for his class of 21 students, Sammy needed two eggs. Now he wants to bake cookies for the upcoming science fair. If he expects 336 people to attend the science fair, how many eggs will he need?

2-113. The graph below shows distances traveled by Car A and Car B. Car A is represented by the line containing point A, and Car B is represented by the line containing point B. Use the graph to answer the following questions.



a. Which car is traveling faster? How can you tell?

b. Find the coordinates of point A and point B.

c. How fast did Car A travel (in miles per hour)? How fast did Car B travel?

2-114. Solve the equations below for x and check your solutions.

a. $-3+x=-2x+6$

b. $5-x=3x+1$

c. $-4x=2x+9$

d. $-(x-3)=-4x$

2-115. Simplify the following expressions.

a. $x+3x-3+2x^2+8x-5$

b. $3y+14y^2-6y^2-9y+1-y-3y$

c. $2y^2+30xy-2y^2+4y-4x$

d. $x-0.2x$

2-116. Mr. Wallis has done it again! He has started to create more tables to help him figure out things like how many gallons of gas it takes to travel a certain number of miles or how many minutes it takes to walk a certain number of blocks. Use proportional reasoning to complete his tables below.

a.

# of Books	Days
2	10
10	50
	60
3	
1	
$\frac{1}{5}$	
	365

b.

Minutes	Blocks
10	25
5	12.5
1	
20	
30	
	0
	35

c.

Miles	Gallons
280	14
140	7
	21
20	
100	
1000	
	17.5

Chapter 2 Closure What have I learned?

Reflection and Synthesis

The activities below offer you a chance to reflect on what you have learned during this chapter. As you work, look for concepts that you feel very comfortable with, ideas that you would like to learn more about, and topics you need more help with. Look for **connections** between ideas as well as **connections** with material you learned previously.

① TEAM BRAINSTORM

With your team, brainstorm a list for each of the following three topics. Be as detailed as you can. How long can you make your list? Challenge yourselves. Be prepared to share your team's ideas with the class.

Topics: What have you studied in this chapter? What ideas and words were important in what you learned? Remember to be as detailed as you can.

Problem Solving: What did you do to solve problems? What different strategies did you use?

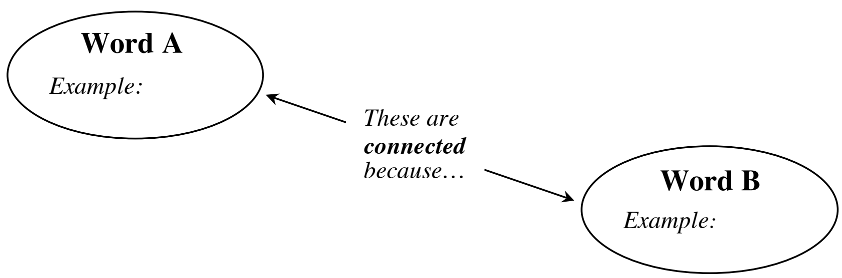
Connections: What topics, ideas, and words that you learned *before* this chapter are **connected** to the new ideas in this chapter? Again, make your list as long as you can.

② MAKING CONNECTIONS

The following is a list of the vocabulary used in this chapter. The words that appear in bold are new to this chapter. Make sure that you are familiar with all of these words and know what they mean. Refer to the glossary or index for any words that you do not yet understand.

algebra tiles	area	**combining like terms**
equal	**equation**	**equation mat**
equivalent	**evaluate**	**expression**
expression comparison mat	**expression mat**	**greater**
minus	**negative**	**opposite**
order of operations	**proportional**	**simplify**
solution	**solve**	sum
term	**variable**	**zero**

Make a concept map showing all of the **connections** you can find among the key words and ideas listed above. To show a **connection** between two words, draw a line between them and explain the **connection**, as shown in the example below. A word can be **connected** to any other word as long as there is a **justified connection**. For each key word or idea, provide a sketch of an example.



Your teacher may provide you with vocabulary cards to help you get started. If you use the cards to plan your concept map, be sure either to re-draw your concept map on your paper or to glue the vocabulary cards to a poster with all of the **connections** explained for others to see and understand.

While you are making your map, your team may think of related words or ideas that are not listed above. Be sure to include these ideas on your concept map.

③ SUMMARIZING MY UNDERSTANDING

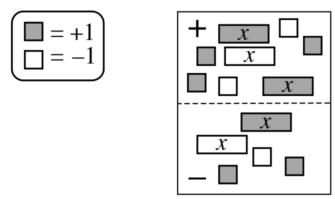
This section gives you an opportunity to show what you know about certain math topics or ideas. Your teacher will give you directions for exactly how to do this. Your teacher may give you a “GO” page to work on. “GO” stands for “Graphic Organizer,” a tool you can use to organize your thoughts and communicate your ideas clearly.

④ WHAT HAVE I LEARNED?

This section will help you evaluate which types of problems you have seen with which you feel comfortable and those with which you need more help. This section appears at the end of every chapter to help you check your understanding. Even if your teacher does not assign this section, it is a good idea to try the problems and find out for yourself what you know and what you need to work on.

Solve each problem as completely as you can. The table at the end of this closure section has answers to these problems. It also tells you where you can find additional help and practice on problems like these.

CL 2-117. Examine the expression mat at right.



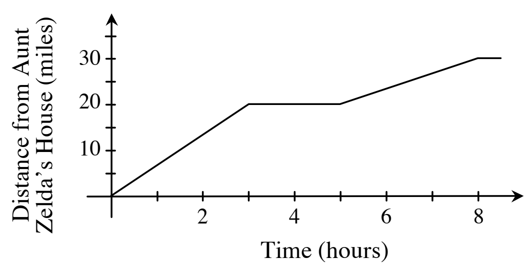
a. Copy the expression mat onto your paper.

b. Write an expression for the tiles as they appear.

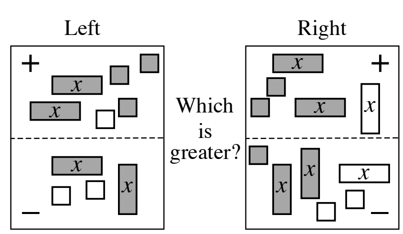
c. On your drawing, circle all of the zeros that you can find to simplify the expression.

d. Write the completely simplified expression.

CL 2-118. Zeke lives 30 miles from his aunt, Zelda, and is riding his bike home from her house. Interpret the graph to tell a story about what could have happened on his ride home.



CL 2-119. Write expressions for each side of the expression comparison mat. Use “legal” moves to simplify and determine which is greater.



CL 2-120. Solve the following problem using Guess and Check. Show your guesses in an organized way.

Ralph and Alphonse are shooting marbles. Ralph has five more marbles than Alphonse, and they have a total of 73 marbles. How many marbles does each of them have?

CL 2-121. Simplify each expression with or without algebra tiles. Record your steps.

a. $3+7x-(2+9x)$

b. $6-(3x-4)+7x-11$

CL 2-122. Copy the pattern below onto graph paper. Draw Figures 1 and 5 on your paper.

a. How many tiles are in each figure?

b. How is the pattern changing?

c. How many tiles would Figure 6 have?

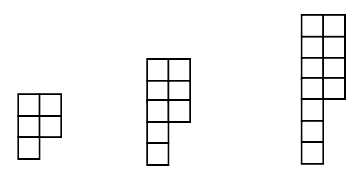


Figure 2 Figure 3 Figure 4

CL 2-123. Silvia has a picture from her trip to the Grand Canyon. The photo is 4 inches tall by 6 inches wide.

a. She would like to make a larger photo for her wall that is as big as possible. The widest the enlarged photo can be is 48 inches. How tall will the enlarged photo be?

b. Silvia also wants a wallet-sized photo to carry around and show her friends that is 1.5 inches tall. How wide will the wallet-sized photo be?

CL 2-124. Evaluate $6x-(3y+7)-xy$ when $x=5$ and $y=3$.

CL 2-125. Simplify the expression below by combining like terms:

$$3x^2+10-y^2+4x-8x^2-5y-8+y^2+3$$

CL 2-126. Solve this equation to find x: $2-(3x-4)=2x-9$.

CL 2-127. Check your answers using the table at the end of the closure section. Which problems do you feel confident about? Which problems were hard? Use the table to make a list of topics you need help on and a list of topics you need to practice more.

⑤ HOW AM I THINKING?

This course focuses on five different **Ways of Thinking**: reversing thinking, justifying, generalizing, making connections, and applying and extending understanding. These are some of the ways in which you think while trying to make sense of a concept or to solve a problem (even outside of math class). During this chapter, you have probably used each Way of Thinking multiple times without even realizing it!

This closure activity will focus on one of these Ways of Thinking: **justifying**. Read the description of this Way of Thinking at right.

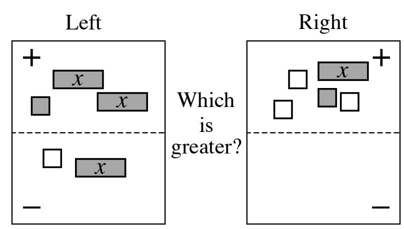
Justifying

You often think this way when you try to convince yourself or someone else that an idea or solution is correct. Often, a justification is the answer to the questions “Why?” or “How do you know for sure?” When you catch yourself thinking, “*I think this is true because…”,* you are justifying.

Think about the topics that you have learned during this chapter. When did you need to convince someone that your thinking was correct? What types of **justification** did you need to use? You may want to flip through the chapter to refresh your memory about the problems that you have worked on. Discuss any ideas you have with the rest of the class.

Once your discussion is complete, analyze how **justifications** work below.

a. While simplifying the expressions shown in the expression comparison mat at right, the four members of a study team made the following statements. Which students **justified** their statements? And were the **justifications** convincing? Explain why or why not.



Rosalita: “I think we can take the positive unit tile and negative unit tile away from the left side because they make zero.”

Anthony: “I think we can take an x-tile away from both sides.”

Barry: “I don’t think we can tell which side is greater because there are more x-tiles on the left side than on the right.”

Deshawn: “I think we can remove a positive and negative unit tile from the “+” region on the right side because they are opposites, so they make zero.”

Continues on next page →

⑤ *Continued from previous page.*

b. Your teammate needs help understanding why $-(-2x-3)=2x+3$. She thinks that $-(-2x-3)=2x-3$. **Justify** why $-(-2x-3)=2x+3$ so that she is convinced.

c. Examine the tile pattern below. What do you think Figure 0 must look like? **Justify** why your Figure 0 fits the pattern.

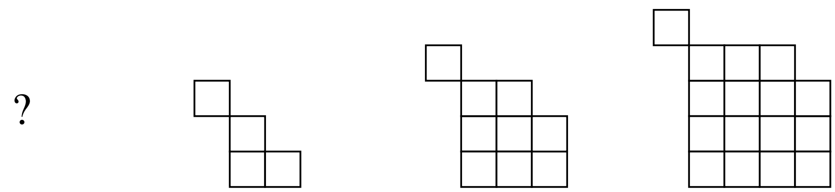
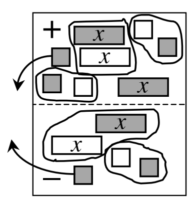
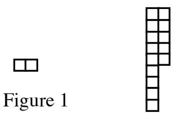


Figure 0 Figure 1 Figure 2 Figure 3

Answers and Support for Closure Activity #4
What Have I Learned?

Problem	Solution	Need Help?	More Practice
CL 2-117.	b. $2x-x+3-2-(x-x+2-1)$ c. one possible answer:  d. x	Problem 2-23, Lesson 2.1.6 Math Notes box	Problems 2-26, 2-36, 2-38, 2-39, 2-42, 2-44, and 2-59
CL 2-118.	There are many possible stories for this graph. Make sure your story explains the changes in speed and the stops and starts in the graph. Zeke travels 20 miles during the first 3 hours and then rests for 2 hours. Then he travels 10 miles in the next 3 hours and reaches his house after 8 hours.	Lesson 1.1.3	Problems 2-98 and 2-113 (also problems 1-6, 1-9, 1-20, 1-47, 1-48, and 1-68)
CL 2-119.	Left: $-1+2x+3-(2x-2)\ =4$ Right: $2x+2-x-(2x-x-2+1)\ =3$ The left expression is greater than the right expression.	Problem 2-48, Lesson 2.1.6 Math Notes box	Problems 2-49, 2-50, 2-57, 2-58, 2-65, 2-66, 2-67, 2-69, 2-73, and 2-77

Problem	Solution	Need Help?	More Practice
CL 2-120.	Ralph has 39 marbles, and Alphonse has 34 marbles.	Lesson 2.1.7 Math Notes box	Problems 2-11, 2-32, 2-68, 2-78, 2-87, and 2-103
CL 2-121.	a. $-2x+1$ b. $4x-1$	Lesson 2.1.4, Lesson 2.1.6 Math Notes box	Problems 2-38, 2-39, 2-44, and 2-59
CL 2-122.	a.  Figure 1 Figure 5 b. Each figure has three more tiles than the one before it. c. Figure 6 would have 17 tiles.	Lesson 1.1.4	Problem 2-70 (also problems 1-31, 1-32, and 1-67)
CL 2-123.	a. The picture would be 32 inches tall. b. The picture would be 2.25 inches wide.	Lessons 2.2.1 and 2.2.3	Problems 2-91, 2-99, 2-102, 2-106, 2-108, 2-109, 2-110, 2-112, and 2-116
CL 2-124.	$6 \cdot 5-(3 \cdot 3+7)-5 \cdot 3=-1$	Lesson 2.1.3 Math Notes box	Problems 2-10, 2-18, 2-30, 2-56, and 2-90
CL 2-125.	$-5x^2+4x-5y+5$	Lesson 2.1.1, Lesson 2.1.5 Math Notes box	Problems 2-4, 2-17, 2-29, 2-52, 2-79, 2-96, and 2-115
CL 2-126.	$x=3$	Problems 2-74 and 2-75, Lesson 2.1.8 Math Notes box	Problems 2-76, 2-82, 2-83, 2-84, 2-95, 2-105, and 2-114