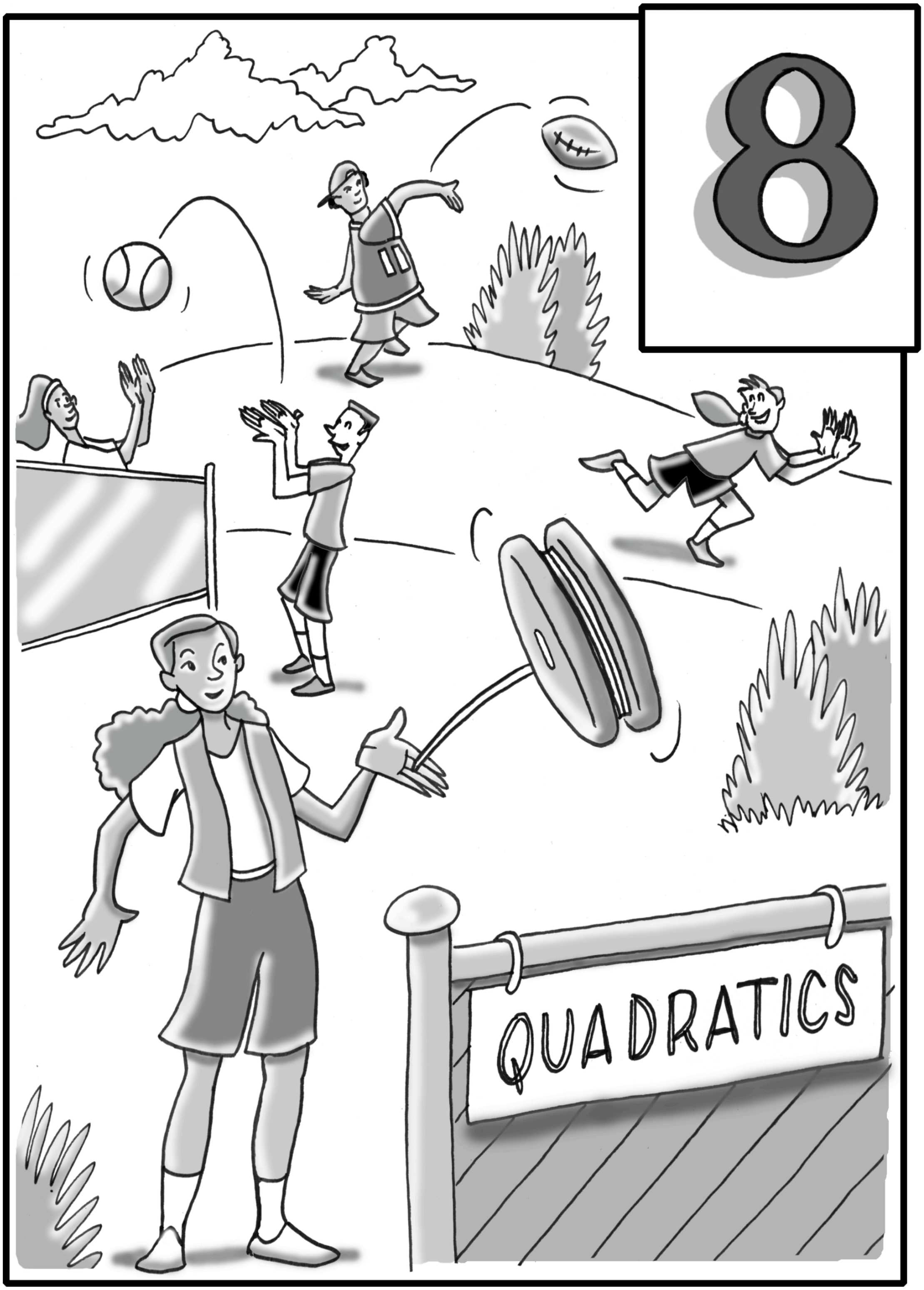

8
QUADRATICS

CHAPTER 8 Quadratics

In Chapter 4, you used a web to organize the connections you found between each of the different representations of lines. These connections enabled you to use any representation (such as a graph, rule, situation, or table) to find any of the other representations.

In this chapter, a quadratics web will challenge you to find connections between the different representations of a parabola. Through this endeavor, you will learn how to rewrite quadratic equations by using a process called factoring. You will also discover and use a very important property of zero.

Guiding Questions

Think about these questions throughout this chapter:

How can I rewrite it?

What's the connection?

What's special about zero?

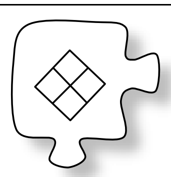
What information do I need?

Is there another method?

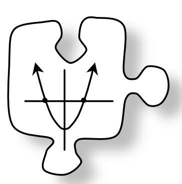
In this chapter, you will learn:

- How to factor a quadratic expression completely.
- How to find the roots of a quadratic equation, if they exist.
- How to move from all representations of a parabola (rule, graph, table, and situation) to each of the other representations directly.

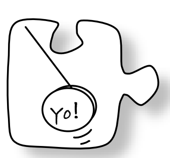
Chapter Outline



Section 8.1 In this section, you will develop a method to change a quadratic equation written as a sum into its product form (also called its factored form).



Section 8.2 Through a fun application, you will find ways to generate each representation of a parabola from each of the others. You will also develop a method to solve quadratic equations using the Zero Product Property.



Section 8.3 In this section, you will be introduced to another method to solve quadratic equations called the Quadratic Formula.

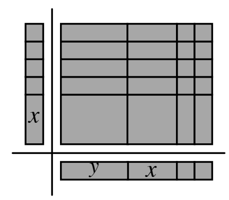
8.1.1 How can I find the product?

Introduction to Factoring Quadratics

In Chapter 5 you learned how to multiply algebraic expressions using algebra tiles and generic rectangles. This section will focus on **reversing** this process: How can you find a product when given a sum?

8-1. Review what you know about products and sums below.

a. Write the area of the rectangle at right as a product and as a sum. Remember that the product represents the area found by multiplying the length by the width, while the sum is the result of adding the areas inside the rectangle.



b. Use a generic rectangle to multiply $(6x-1)(3x+2)$. Write your solution as a sum.

8-2. The process of changing a sum to a product is called **factoring**. Can every expression be factored? That is, *does every sum have a product that can be represented with tiles?*

Investigate this question by building rectangles with algebra tiles for the following expressions. For each one, write the area as a sum and as a product. If you cannot build a rectangle, be prepared to convince the class that no rectangle exists (and thus the expression cannot be factored).

a. $2x^2+7x+6$

b. $6x^2+7x+2$

c. x^2+4x+1

d. $2xy+6x+y^2+3y$

8-3. Work with your team to find the sum and the product for the following generic rectangles. Are there any special strategies you discovered that can help you determine the dimensions of the rectangle? Be sure to share these strategies with your teammates.

a.

$2x$	5
$6x^2$	$15x$

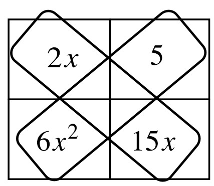
b.

$-2y$	-6
$5xy$	$15x$

c.

$-9x$	-12
$12x^2$	$16x$

8-4. While working on problem 8-3, Casey noticed a pattern with the diagonals of each generic rectangle. However, just before she shared her pattern with the rest of her team, she was called out of class! The drawing on her paper looked like the diagram below. Can you figure out what the two diagonals have in common?



8-5. Does Casey's pattern always work? Verify that her pattern works for all of the 2-by-2 generic rectangles in problem 8-3. Then describe Casey's pattern for the diagonals of a 2-by-2 generic rectangle in your Learning Log. Be sure to include an example. Title this entry "Diagonals of a Generic Rectangle" and include today's date.

MATH NOTES

METHODS AND MEANINGS

New Vocabulary to Describe Algebraic Expressions

Since algebraic expressions come in many different forms, there are special words used to help describe these expressions. For example, if the expression can be written in the form $ax^2 + bx + c$ and if a is not 0, it is called a **quadratic** expression. Study the examples of quadratic expressions below.

Examples of quadratic expressions:

$x^2 - 15x + 26$

$16m^2 - 25$

$12 - 3k^2 + 5k$

The way an expression is written can also be named. When an expression is written in product form, it is described as being **factored**. When factored, each of the expressions being multiplied is called a **factor**. For example, the factored form of $x^2 - 15x + 26$ is $(x - 13)(x - 2)$, so $x - 13$ and $x - 2$ are each factors of the original expression.

Finally, the number of terms in an expression can help you name the expression to others. If the expression has one term, it is called a **monomial**, while an expression with two terms is called a **binomial**. If the expression has three terms, it is called a **trinomial**. Study the examples below.

Examples of monomials: $15xy^2$ and $-2m$

Examples of binomials: $16m^2 - 25$ and $7h^9 + \frac{1}{2}h$

Examples of trinomials: $12 - 3k^2 + 5k$ and $x^2 - 15x + 26$

Review & Preview

8-6. Write the area of the rectangle at right as a sum and as a product.

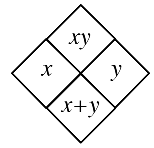
$-3x$	$-6y$	12
$2x^2$	$4xy$	$-8x$

8-7. Multiply the expressions below using a generic rectangle. Then verify Casey's pattern (that the product of one diagonal equals the product of the other diagonal).

a. $(4x-1)(3x+5)$

b. $(2x-7)^2$

8-8. Remember that a Diamond Problem is a pattern for which the **product** of two numbers is placed on top, while the **sum** of the same two numbers is placed on bottom. (This pattern is demonstrated in the diamond at right.) Copy and complete each Diamond Problem below.



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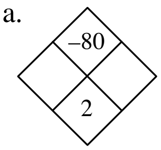

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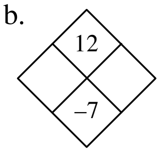

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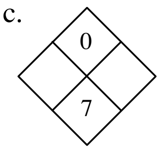

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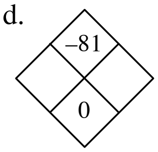

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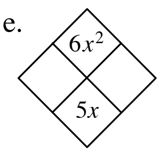

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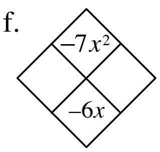


8-9. For each line below, name the slope and y-intercept.

a. $y=\frac{-1+4x}{2}$

b. $3x+y=-7$

c. $y=\frac{-2}{3}x+8$

d. $y=-2$

8-10. On graph paper, graph $y=x^2-2x-8$.

a. Name the y-intercept. What is the connection between the y-intercept and the rule $y=x^2-2x-8$?

b. Name the x-intercepts.

c. Find the lowest point of the graph, the vertex.

8-11. Calculate the value of each expression below.

a. $5-\sqrt{36}$

b. $1+\sqrt{39}$

c. $-2-\sqrt{5}$

8.1.2 Is there a shortcut?

Factoring with Generic Rectangles

Since mathematics is often described as the study of patterns, it is not surprising that generic rectangles have many patterns. You saw one important pattern in Lesson 8.1.1 (Casey's pattern from problem 8-4). Today you will continue to use patterns while you develop a method to factor trinomial expressions.

8-12. Examine the generic rectangle shown at right.

$-35x$	14
$10x^2$	$-4x$

a. Review what you learned in Lesson 8.1.1 by writing the area of the rectangle at right as a sum and as a product.

b. Does this generic rectangle fit Casey's pattern for diagonals? Demonstrate that the product of each diagonal is equal.

8-13. FACTORING QUADRATICS

To develop a method for factoring without algebra tiles, first study how to factor with algebra tiles, and then look for connections within a generic rectangle.

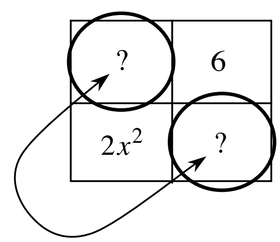
a. Using algebra tiles, factor $2x^2 + 5x + 3$; that is, use the tiles to build a rectangle, and then write its area as a product.

b. To factor with tiles (like you did in part (a)), you need to determine how the tiles need to be arranged to form a rectangle. Using a generic rectangle to factor requires a different process.

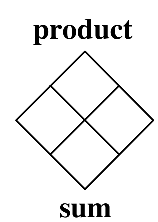
Miguel wants to use a generic rectangle to factor $3x^2 + 10x + 8$. He knows that $3x^2$ and 8 go into the rectangle in the locations shown at right. Finish the rectangle by deciding how to place the ten x-terms. Then write the area as a product.

	8
$3x^2$	

c. Kelly wants to find a shortcut to factor $2x^2 + 7x + 6$. She knows that $2x^2$ and 6 go into the rectangle in the locations shown at right. She also remembers Casey's pattern for diagonals. Without actually factoring yet, what do you know about the missing two parts of the generic rectangle?



d. To complete Kelly's generic rectangle, you need two x-terms that have a sum of $7x$ and a product of $12x^2$. Create and solve a Diamond Problem that represents this situation.

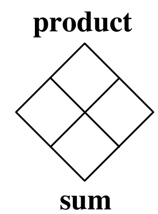


e. Use your results from the Diamond Problem to complete the generic rectangle for $2x^2 + 7x + 6$, and then write the area as a product of factors.

8-14. Factoring with a generic rectangle is especially convenient when algebra tiles are not available or when the number of necessary tiles becomes too large to manage. Using a Diamond Problem helps avoid guessing and checking, which can at times be challenging. Use the process from problem 8-13 to factor $6x^2 + 17x + 12$. The questions below will guide your process.

a. When given a trinomial, such as $6x^2 + 17x + 12$, what two parts of a generic rectangle can you quickly complete?

b. How can you set up a Diamond Problem to help factor a trinomial such as $6x^2 + 17x + 12$? What goes on the top? What goes on the bottom?



c. Solve the Diamond Problem for $6x^2 + 17x + 12$ and complete its generic rectangle.

d. Write the area of the rectangle as a product.

8-15. Use the process you developed in problem 8-13 to factor the following quadratics, if possible. If a quadratic cannot be factored, justify your conclusion.

a. $x^2 + 9x + 18$

b. $4x^2 + 17x - 15$

c. $4x^2 - 8x + 3$

d. $3x^2 + 5x - 3$

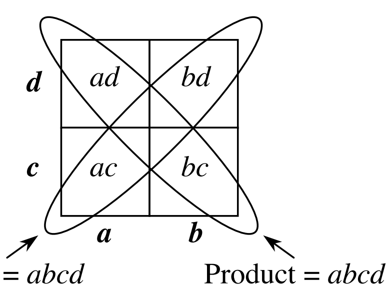
MATH NOTES

METHODS AND MEANINGS

Diagonals of Generic Rectangles

Why does Casey's pattern from problem 8-4 work? That is, why does the product of the terms in one diagonal of a 2-by-2 generic rectangle always equal the product of the terms in the other diagonal?

Examine the generic rectangle at right for $(a + b)(c + d)$. Notice that each of the resulting diagonals have a product of *abcd*. Thus, the product of the terms in the diagonals are equal.



Review & Preview

8-16. Use the process you developed in problem 8-13 to factor the following quadratics, if possible.

a. $x^2 - 4x - 12$

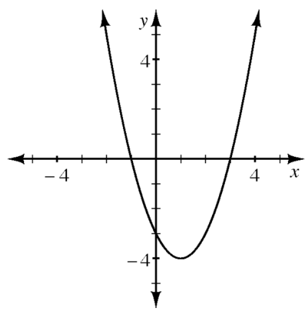
b. $4x^2 + 4x + 1$

c. $2x^2 - 9x - 5$

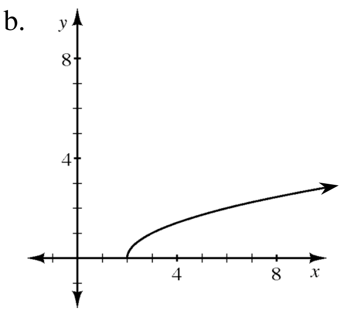
d. $3x^2 + 10x - 8$

8-17. For each rule represented below, state the x- and y-intercepts, if possible.

a.



b.



c.

x	–5	–4	–3	–2	–1	0	1	2
y	8	4	0	–4	0	2	0	–4

d. $5x - 2y = 40$

8-18. Graph $y = x^2 - 9$ on graph paper.

a. Name the y-intercept. What is the connection between the y-intercept and the rule $y = x^2 - 9$?

b. Name the x-intercepts. What is the connection between the x-intercepts and the rule $y = x^2 - 9$?

8-19. Find the point of intersection for each system.

a. $y = 2x - 3$
$x + y = 15$

b. $3x = y - 2$
$6x = 4 - 2y$

8-20. Solve each equation below for the given variable, if possible.

a. $\frac{4x}{5} = \frac{x-2}{7}$

b. $-3(2b - 7) = -3b + 21 - 3b$

c. $6 - 2(c - 3) = 12$

8-21. Find the equation of the line that passes through the points (–800, 200) and (–400, 300).

8.1.3 How can I factor this?

Factoring with Special Cases

Practice your new method for factoring quadratic expressions without tiles as you consider special types of quadratic expressions.

8-22. Factor each quadratic below, if possible. Use a Diamond Problem and generic rectangle for each one.

a. $x^2 + 6x + 9$

b. $2x^2 + 5x + 3$

c. $x^2 + 5x - 7$

d. $3m^2 + m - 14$

8-23. SPECIAL CASES

Most quadratics are written in the form $ax^2 + bx + c$. But what if a term is missing? Or what if the terms are in a different order? Consider these questions while you factor the expressions below. Share your ideas with your teammates and be prepared to demonstrate your process for the class.

a. $9x^2 - 4$

b. $12x^2 - 16x$

c. $3 + 8k^2 - 10k$

d. $40 - 100m$

8-24. Now turn your attention to the quadratic below. Use a generic rectangle and Diamond Problem to factor this expression. Compare your answer with your teammates' answers. Is there more than one possible answer?

$$4x^2 - 10x - 6$$

8-25. The multiplication table below has factors along the top row and left column. Their product is where the row and column intersect. With your team, complete the table with all of the factors and products.

Multiply	$x-2$	
$x+7$		
	$3x^2-5x-2$	$6x^2+5x+1$

8-26. In your Learning Log, explain how to factor a quadratic expression. Be sure to offer examples to demonstrate your understanding. Include an explanation of how to deal with special cases, such as when a term is missing or when the terms are not in standard order. Title this entry "Factoring Quadratics" and include today's date.

METHODS AND MEANINGS

MATH NOTES

Standard Form of a Quadratic

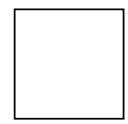
A quadratic expression in the form ax^2+bx+c is said to be in **standard form**. Notice that the terms are in order from greatest exponent to least.

Examples of quadratic expressions in standard form: $3m^2+m-1$, x^2-9, and $3x^2+5x$. Notice that in the second example, $b = 0$, while in the third example, $c = 0$.

Review & Preview

8-27. The perimeter of a triangle is 51 cm. The longest side is twice the length of the shortest side. The third side is 3 cm longer than the shortest side. How long is each side? Write an equation that represents the problem and then solve it.

8-28. Remember that a square is a rectangle with four equal sides.



a. If a square has an area of 81 square units, how long is each side?

b. Find the length of the side of a square with area 225 square units.

c. Find the length of the side of a square with area 10 square units.

d. Find the area of a square with side 11 units.

8-29. Factor the following quadratics, if possible.

a. $k^2 - 12k + 20$

b. $6x^2 + 17x - 14$

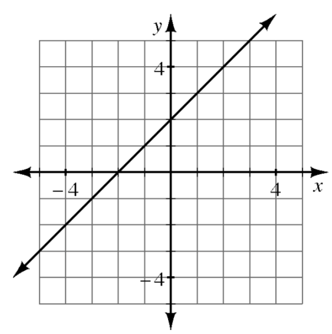
c. $x^2 - 8x + 16$

d. $9m^2 - 1$

8-30. Examine the two equations below. Where do they intersect?

$$y = 4x - 3$$
$$y = 9x - 13$$

8-31. Find the equation of a line perpendicular to the one graphed at right that passes through the point (6, 2).



8-32. Solve each equation below for *x*. Check each solution.

a. $2x - 10 = 0$

b. $x + 6 = 0$

c. $(2x - 10)(x + 6) = 0$

d. $4x + 1 = 0$

e. $x - 8 = 0$

f. $(4x + 1)(x - 8) = 0$

8.1.4 Can it still be factored?

Factoring Completely

There are many ways to write the number 12 as a product of factors. For example, 12 can be rewritten as $3 \cdot 4$, as $2 \cdot 6$, as $1 \cdot 12$, or as $2 \cdot 2 \cdot 3$. While each of these products is accurate, only $2 \cdot 2 \cdot 3$ is considered to be **factored completely**, since the factors are prime and cannot be factored themselves.

During this lesson you will learn more about what it means for a quadratic expression to be factored completely.

8-33. Review what you have learned by factoring the following expressions, if possible.

a. $9x^2 - 12x + 4$

b. $81m^2 - 1$

c. $28 + x^2 - 11x$

d. $3n^2 + 9n + 6$

8-34. Compare your solutions for problem 8-33 with the rest of your class.

a. Is there more than one factored form of $3n^2 + 9n + 6$? Why or why not?

b. Why does $3n^2 + 9n + 6$ have more than one factored form while the other quadratics in problem 8-33 only have one possible answer? Look for clues in the original expression ($3n^2 + 9n + 6$) and in the different factored forms.

c. **Without factoring**, predict which quadratic expressions below may have more than one factored form. Be prepared to defend your choice to the rest of the class.

i. $12t^2 - 10t + 2$

ii. $5p^2 - 23p - 10$

iii. $10x^2 + 25x - 15$

iv. $3k^2 + 7k - 6$

8-35. FACTORING COMPLETELY

In part (c) of problem 8-34, you should have noticed that each term in $12t^2 - 10t + 2$ is divisible by 2. That is, it has a **common factor** of 2.

a. What is the common factor for $10x^2 + 25x - 15$?

b. For an expression to be **completely factored**, each factor must have all common factors separated out. Sometimes it is easiest to do this first. Since 5 is a common factor of $10x^2 + 25x - 15$, you can factor $10x^2 + 25x - 15$ using a special generic rectangle, which is shown below. Find the length of this generic rectangle and write its area as a product of its length and width.

5	$10x^2$	$+25x$	-15

c. Can the result be factored even more? That is, can either factor from the result from part (b) above also be factored? Factor any possible expressions and write your solution as a product of all three factors.

8-36. Factor each of the following expressions as completely as possible.

a. $5x^2 + 15x - 20$

b. $3x^3 - 6x^2 - 45x$

c. $2x^2 - 50$

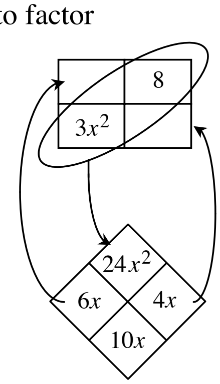
d. $x^2y - 3xy - 10y$

MATH NOTES

METHODS AND MEANINGS

Factoring Quadratic Expressions

Review the process of factoring quadratics developed in problem 8-13 and outlined below. This example demonstrates how to factor $3x^2 + 10x + 8$.



1. Place the x^2- and constant terms of the quadratic expression in opposite corners of the generic rectangle. Determine the sum and product of the two remaining corners: The sum is simply the x-term of the quadratic expression, while the product is equal to the product of the x^2- and constant terms.
2. Place this sum and product into a Diamond Problem and solve it.
3. Place the solutions from the Diamond Problem into the generic rectangle and find the dimensions of the generic rectangle.
4. Write your answer as a product: $(3x + 4)(x + 2)$.

2	$6x$	8
x	$3x^2$	$4x$
	$3x$	4

Review & Preview

8-37. Factor the quadratic expressions below. If the quadratic is not factorable, explain why not.

a. $2x^2 + 3x - 5$

b. $x^2 - x - 6$

c. $3x^2 + 13x + 4$

d. $2x^2 + 5x + 7$

8-38. A line has intercepts (4, 0) and (0, –3). Find the equation of the line.

8-39. As Jhalil and Joman practice for the SAT, their scores on practice tests rise. Jhalil's current score is 850, and it is rising by 10 points per week. On the other hand, Joman's current score is 570 and is growing by 50 points per week.

a. When will Joman's score catch up to Jhalil's?

b. If the SAT test is in 12 weeks, who will score highest?

8-40. Mary says that you can find an *x*-intercept by substituting 0 for *x*, while Michelle says that you need to substitute 0 for *y*.

a. Who, if anyone, is correct and why?

b. Use the correct approach to find the *x*-intercept of $-4x + 5y = 16$.

8-41. Find three consecutive numbers whose sum is 138 by writing and solving an equation.

8-42. Match each rule below with its corresponding graph. Can you do this without making any tables? Explain your selections.

a. $y = -x^2 - 2$

b. $y = x^2 - 2$

c. $y = -x^2 + 2$

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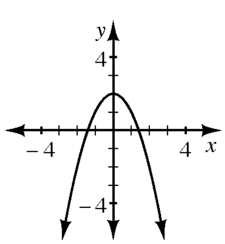

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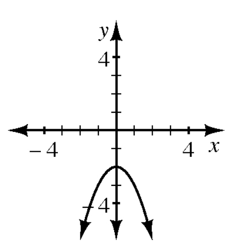

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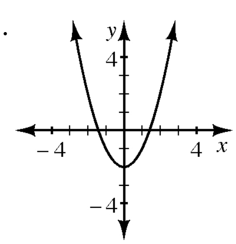
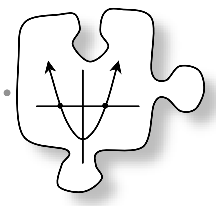


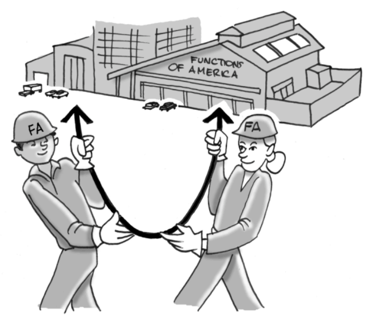
8.2.1 What do I know about a parabola?



Investigating a Parabola

In previous chapters, you have investigated linear equations. In Section 8.2, you will study parabolas. You will learn all you can about their shape, study different equations used to graph them, and see how they can be used in real-life situations.

8-43. FUNCTIONS OF AMERICA



Congratulations! Your work at the Line Factory was so successful that the small local company grew into a national corporation called Functions of America. Recently your company has had some growing pains, and your new boss has turned to your team for help. See her memo below.

MEMO

To: Your study team
From: Ms. Freda Function, CEO
Re: New product line

I have heard that while lines are very popular, there is a new craze in Europe to have non-linear designs. I recently visited Paris and Milan and discovered that we are behind the times!

Please investigate a new function called a parabola. I'd like a full report at the end of today with any information your team can give me about its shape and equation. Spare no detail! I'd like to know everything you can tell me about how the rule for a parabola affects its shape. I'd also like to know about any special points on a parabola or any patterns that exist in its table.

Remember, the company is only as good as its employees! I need you to uncover the secrets that our competitors do not know.

Sincerely,
Ms. Function, CEO

Problem continues on next page →

8-43. *Problem continued from previous page.*

Your Task: Your team will be assigned its own parabola to study. Investigate your team's parabola and be ready to describe everything you can about it by using its graph, rule, and table. Answer the questions below to get your investigation started. You may answer them in any order; however, do not limit yourselves to these questions!

- Does your parabola have any **symmetry**? That is, can you fold the graph of your parabola so that each side of the fold exactly matches the other? If so, where would the fold be? Do you think this works for all parabolas? Why or why not?
- Is there a highest or lowest point on the graph of your parabola? If so, where is it? This point is called a **vertex**. How can you describe the parabola at this point?
- Are there any special points on your parabola? Which points do you think are important to know? Are there any special points that you expected but do not exist for your parabola? What connection(s) do these points have with the rule of your parabola?
- How would you describe the shape of your parabola? For example, would you describe your parabola as pointing up or down? Do the sides of the parabola ever go straight up or down (vertically)? Why or why not? Is there anything else special about its shape?

List of Parabolas:

$y = x^2 - 2x - 8$	$y = -x^2 + 4$
$y = x^2 - 4x + 5$	$y = x^2 - 2x + 1$
$y = x^2 - 6x + 5$	$y = -x^2 + 3x + 4$
$y = -x^2 + 2x - 1$	$y = x^2 + 5x + 1$

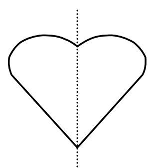
8-44. Prepare a poster for the CEO detailing your findings from your parabola investigation. Include any insights you and your teammates found. Explain your conclusions and justify your statements. Remember to include a complete graph of your parabola with all special points carefully labeled.

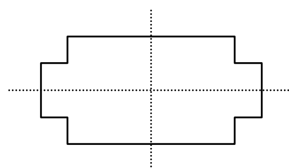
MATH NOTES

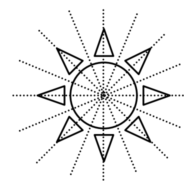
LOOKING DEEPER

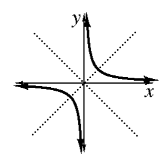
Symmetry

When a graph or picture can be folded so that both sides of the fold will perfectly match, it is said to have **symmetry**. The line where the fold would be is called the **line of symmetry**. Some shapes have more than one line of symmetry. See the examples below.


This shape has one line of symmetry.


This shape has two lines of symmetry.


This shape has eight lines of symmetry.



This graph has two lines of symmetry.

Review & Preview

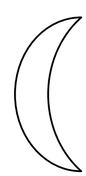
8-45. Calculate the value of each expression below.

a. $\frac{2+\sqrt{16}}{3}$ b. $\frac{-1+\sqrt{49}}{-2}$ c. $\frac{-10-\sqrt{5}}{2}$

8-46. Find the equation of the line that goes through the points (–15, 70) and (5, 10).

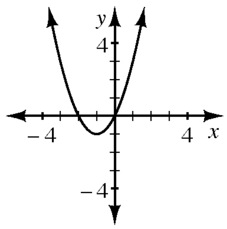
8-47. Change $6x-2y=10$ to slope-intercept ($y=mx+b$) form. Then state the slope (m) and the y-intercept (b).

8-48. Copy the figure at right onto your paper. Then draw any lines of symmetry.

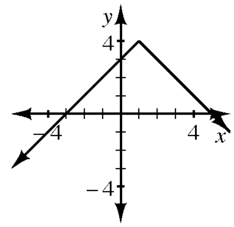


8-49. For each rule represented below, state the x- and y-intercepts.

a.



b.



c.

x	–3	–2	–1	0	1	2	3
y	8	3	0	–1	0	3	8

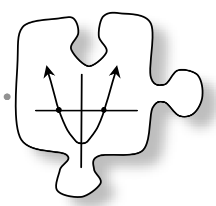
d. $2x + 3y = 18$

8-50. Use a generic rectangle to multiply each expression below.

a. $(3x - 4)(2x + 3)$

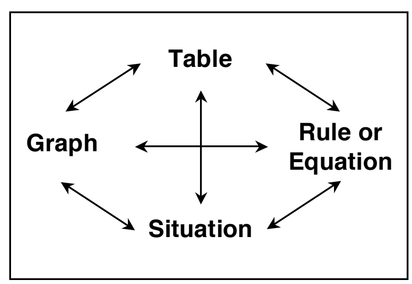
b. $(5x - 2)^2$

8.2.2 What's the connection?



Multiple Representations for Quadratics

In Chapter 4 you completed a web for the different representations of linear equations. You discovered special shortcuts to help you move from one representation to another. For example, given a linear equation, you can now draw the corresponding graph as well as determine an equation from a graph.



Today you will explore the connections between the different representations for quadratics. As you work, keep in mind the following questions:

What representations are you using?

What is the connection between the various representations?

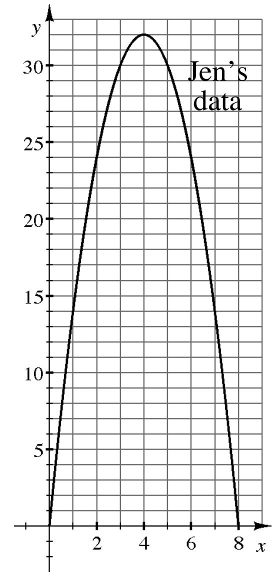
What do you know about a parabola?

8-51. WATER-BALLOON CONTEST

Every year Newtown High School holds a water-balloon competition during halftime of their homecoming game. Each contestant uses a catapult to launch a water balloon from the ground on the football field. This year you are the judge! You must decide which contestants win the prizes for *Longest Distance* and *Highest Launch*. Fortunately, you have a computer that will collect data for each throw. The computer uses x to represent horizontal distance in yards from the goal line and y to represent the height in yards.

The announcer shouts, "Maggie Nanimos, you're up first!" She runs down and places her catapult at the 3-yard line. After Maggie's launch, the computer reports that the balloon traveled along the parabola $y=-x^2+17x-42$.

Then you hear, "Jen Erus, you're next!" Jen runs down to the field, places her catapult at the goal line, and releases the balloon. The tracking computer reports the path of the balloon with the graph at right.

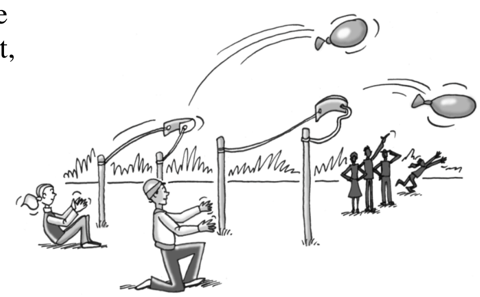


The third contestant, Imp Ecable, accidentally launches the balloon before you are ready. The balloon launches, you hear a roar from the crowd, turn around, and…SPLAT! The balloon soaks you and your computer! You only have time to write down the following partial information about the balloon's path before your computer fizzles:

x (yards)	2	3	4	5	6	7	8	9
y (yards)	0	9	16	21	24	25	24	21

Finally, the announcer calls for the last contestant, Al Truistic. With your computer broken, you decide to record the balloon's height and distance by hand. Al releases the balloon from the 10-yard line. The balloon reaches a height of 27 yards and lands at the 16-yard line.

a. Obtain the Lesson 8.2.2 Resource Page from your teacher. For each contestant, create a table and graph using the information provided for each toss. Determine which of these contestants should win the *Longest Distance* and *Highest Throw* contests.



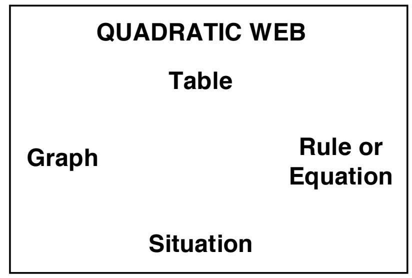
b. Find the x-intercepts of each parabola. What information do the x-intercepts tell you about each balloon toss?

c. Find the vertex of each parabola. What information does the vertex tell you about each balloon throw?

8-52. Today you have explored the four different representations of quadratics: table, graph, equation, and a description of a physical situation involving motion. Draw the representations of the web as shown below in your Learning Log and label it "Quadratic Web."

a. Draw in arrows showing the connections that you currently know how to make between different representations. Be prepared to **justify** a connection for the class.

b. What connections are still missing?



8-53. SITUATION TO RULE

Review how to write a rule from a situation by examining the tile pattern below.

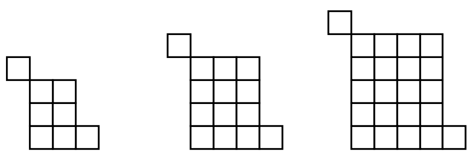
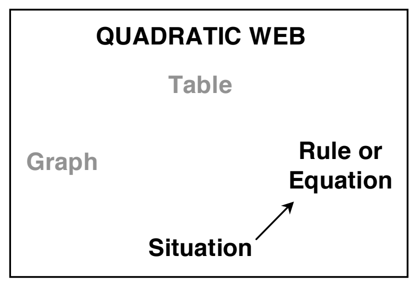


Figure 1 Figure 2 Figure 3



a. Write a rule to represent the number of tiles in Figure x.

b. Is the rule from part (a) **quadratic**? Explain how you know.

c. If you have not done so already, add this pathway to your web from problem 8-52.

8-54. Graph $y = x^2 - 8x + 7$ and label its vertex, x-intercepts, and y-intercepts.

8-55. What is special about the number zero? Think about this as you answer the questions below.

a. Find each sum:

$0+3=$ $\qquad -7+0=$ $\qquad 0+6=$ $\qquad 0+(-2)=$

b. What is special about adding zero? Write a sentence that begins, "When you add zero to a number, …"

c. Julia is thinking of two numbers *a* and *b*. When she adds them together, she gets a sum of *b*. Does that tell you anything about either of Julia's numbers?

d. Find each product:

$3\cdot 0=$ $\qquad (-7)\cdot 0=$ $\qquad 0\cdot 6=$ $\qquad 0\cdot(-2)=$

e. What is special about multiplying by zero? Write a sentence that begins, "When you multiply a number by zero, …"

8-56. Based on the tables below, say as much as you can about the *x*- and *y*-intercepts of the corresponding graphs.

a.

x	y
2	0
0	18
–4	0
–1	–8
6	22
3	0

b.

x	y
7	–4
3	0
10	8
0	–3
8	0
–7	–1

c.

x	y
0	–4
–5	11
3	–2
1	0
13	27
–6	14

8-57. For the line described by the equation $y=2x+6$:

a. What is the *x*-intercept?

b. What is the slope of any line perpendicular to the given line?

8-58. Solve the following systems of equations using any method. Check your solution if possible.

a. $6x-2y=10$
$3x-y=2$

b. $x-3y=1$
$y=16-2x$

8.2.3 How are quadratic rules and graphs connected?

Zero Product Property

You already know a lot about quadratics and parabolas, and you have made several connections between their different representations on the quadratic web. Today you are going to develop a method to sketch a parabola from its equation without a table.

QUADRATIC WEB

Table

Graph ← Rule or Equation

Situation

8-59. WHAT DO YOU NEED TO SKETCH A PARABOLA?

How many points do you need in order to sketch a parabola? 1? 10? 50? Think about this as you answer the questions below. (Note: A sketch does not need to be exact. The parabola merely needs to be reasonably placed with important points clearly labeled.)

a. Can you sketch a parabola if you only know where its y-intercept is? For example, if the y-intercept of a parabola is at (0, –15), can you sketch its graph? Why or why not?

b. What about the two x-intercepts of the parabola? If you only know where the x-intercepts are, can you draw the parabola? For example, if the x-intercepts are at (–3, 0) and (5, 0), can you predict the path of the parabola?

c. Can you sketch a parabola with only its x-intercepts and y-intercept? To test this idea, sketch the graph of a parabola $y = x^2 - 2x - 15$ with x-intercepts (–3, 0) and (5, 0) and y-intercept (0, –15).

8-60. In problem 8-59, you learned that if you can find the intercepts of a parabola from a rule, then you can sketch its graph without a table.

a. What is true about the value of y for all x-intercepts? What is true about the value of x for all y-intercepts? Review your knowledge of intercepts and describe it here.

b. If $x = 0$ at the y-intercept, find the y-intercept of the graph of $y = 2x^2 + 5x - 12$.

c. Since the x-intercept occurs when $y = 0$, write the equation that you would need to solve to find the x-intercepts of the graph of $y = 2x^2 + 5x - 12$.

d. The solutions of the equation $2x^2 + 5x - 12 = 0$ are called its **roots** and are the **zeros** of $2x^2 + 5x - 12$. At this point, can you solve $2x^2 + 5x - 12 = 0$ for x? Explain why or why not.

8-61. ZERO PRODUCT PROPERTY

The equation you wrote in part (c) of problem 8-60 is called a **quadratic equation**. To solve it, you need to examine what you know about zero. Study the special properties of zero below.

Nathan, Nancy, and Gaston are playing a game where Nathan and Nancy each think of a number and then give Gaston a clue about their numbers. Using the clue, Gaston must tell them everything that he knows about their numbers.

a. Nathan and Nancy's first clue for Gaston is that when you multiply their numbers together, the result is zero. What conclusion can Gaston make?

b. Disappointed that Gaston came so close to figuring out their numbers, Nathan and Nancy invite Nadia over to make things harder. Nathan, Nancy, and Nadia all think of secret numbers. This time Gaston is told that when their *three* secret numbers are multiplied together, the answer is zero. What can Gaston conclude this time?

c. Does it matter how many numbers are multiplied? If the product is zero, what do you know about one of the numbers? This property is called the **Zero Product Property**. With the class, write a description of this property in your Learning Log. Title this entry "Zero Product Property" and include today's date.

8-62. How can you use the Zero Product Property to help you solve the quadratic equation $0 = 2x^2 + 5x - 12$ from part (c) of problem 8-60?

a. Examine the quadratic equation. Is there a product that equals zero? If not, how can you rewrite the quadratic expression as a product?

b. Now that the equation is written as a product of factors equaling zero, you can use the Zero Product Property to solve it. Since you know that one of the factors must be zero, you can set up two smaller equations to help you solve for x. Use one factor at a time and determine what x-value makes it equal to zero.

c. What do these solutions represent? What do they tell you?

d. You now know the roots of the equation $0 = 2x^2 + 5x - 12$ (also called the zeros of $2x^2 + 5x - 12$). Use the roots to find the x-intercepts of the graph of the parabola $y = 2x^2 + 5x - 12$. Then sketch a graph of the parabola.

8-63. Use a similar process to sketch the parabola $y = x^2 + x - 6$ by using its intercepts.

8-64. Sketch the parabola $y = 2x^2 + 6x + 4$ by using its intercepts.

MATH NOTES

METHODS AND MEANINGS

Zero Product Property

When the product of two or more numbers is zero, one of those numbers must be zero. This is known as the **Zero Product Property**. If the two numbers are *a* and *b*, this property can be written as follows:

If *a* and *b* are two numbers where $a \cdot b = 0$, then $a = 0$ or $b = 0$.

For example, if $(2x-3)(x+5) = 0$, then $2x - 3 = 0$ or $x + 5 = 0$. Solving yields the solutions $x = \frac{3}{2}$ or $x = -5$. This property helps you solve quadratic equations when the equation can be written as a product of factors.

Review & Preview

8-65. Compare the two equations below.

$$(x+2)(x-1) = 0 \text{ and } (x+2)+(x-1) = 0$$

a. How are the equations different?

b. Solve both equations.

8-66. For each equation below, solve for *x*.

a. $(x-2)(x+8) = 0$

b. $(3x-9)(x-1) = 0$

c. $(x+10)(2x-5) = 0$

d. $(x-7)^2 = 0$

8-67. Examine the system of equations below.

$$5x - 2y = 4$$
$$x = 0$$

a. Before solving this system, Danielle noticed that the point of intersection is also the y-intercept of $5x - 2y = 4$. Explain how she knows this.

b. Find the point of intersection of the two rules above.

8-68. The x-intercepts of the graph of $y = 2x^2 - 16x + 30$ are (3, 0) and (5, 0).

a. What is the x-coordinate of the vertex? How do you know?

b. Use your answer to part (a) above to find the y-coordinate of the vertex. Then write the vertex as a point (x, y).

8-69. Factor each quadratic below completely.

a. $2x^2 - 2x - 4$

b. $4x^2 - 24x + 36$

8-70. The "≤" symbol represents "less than or equal to," while the "<" symbol represents "less than."

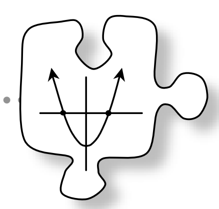
a. Similarly, translate "≥" and ">."

b. How can you write an expression that states that 5 is greater than 3?

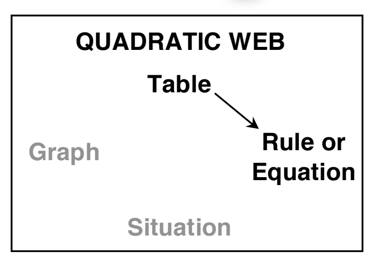
c. Write another expression that states that x is less than or equal to 9.

d. Translate the expression $-2 < 7$ into words.

8.2.4 What new connection can I make?



Solving Quadratic Equations by Factoring



In Lesson 8.2.3, you developed a method for finding the x-intercepts of a parabola given by $y = ax^2 + bx + c$ by finding the roots of the corresponding quadratic equation, $ax^2 + bx + c = 0$, or the zeros of $ax^2 + bx + c$. Today you will learn how to use that skill to solve a wide variety of quadratic equations.

You will also revisit the quadratic web, make a connection between the table and rule of a parabola, and then apply this connection to the water-balloon competition you analyzed in problem 8-51.

8-71. Review what you learned in Lesson 8.2.3 by sketching the graph of $y = x^2 + 3x + 2$ without a table. Specifically, find the x-intercepts and the y-intercept of the parabola and sketch its graph.

8-72. Part of finding the x-intercepts of a parabola involves creating a quadratic equation of the form $ax^2 + bx + c = 0$ and finding its roots (which are also the zeros of the expression $ax^2 + bx + c$). Practice using the Zero Product Property to solve the quadratic equations below.

a. $x^2 + 6x + 8 = 0$

b. $0 = 3x^2 - 7x + 4$

c. $(x + 5)(-2x + 3) = 0$

d. $x^2 + 6x = 0$

e. $0 = 3(x - 5)(2x + 3)$

f. $x^2 + 4x - 9 = 3$

8-73. TABLE TO RULE

You know how to make a table for a quadratic rule, but how can you write an equation when given the table? Examine this new connection that requires you to **reverse** your understanding of the Zero Product Property as you find a rule for each table below. What clues in the tables helped you find the rule?

a.

x	–4	–3	–2	–1	0	1	2	3	4
y	6	0	–4	–6	–6	–4	0	6	14

b.

x	–6	–5	–4	–3	–2	–1	0	1	2	3	4
y	7	0	–5	–8	–9	–8	–5	0	7	16	27

8-74. WATER-BALLOON CONTEST REVISITED

Remember Imp's water-balloon toss? Since the water balloon was thrown on the computer, you were given only a table of data, shown again below. Find a rule that represents the height of Imp's balloon as it traveled through the air.

x (yards)	2	3	4	5	6	7	8	9
y (yards)	0	9	16	21	24	25	24	21

8-75. Find the quadratic web in your Learning Log entry from Lesson 8.2.3. In this entry, add a short explanation for how to find a quadratic equation from its table. Then add an arrow to your web for the connection you made today.

Review & Preview

8-76. Jamie was given the problem, "Find the result when the factors of $65x^2+212x-133$ are multiplied together." Before she could answer, her sister, Lauren, said, "I know the answer without factoring or multiplying!" What was Lauren's answer and how did she know?

8-77. Solve the equations below for x. Check your solutions.

a. $(6x-18)(3x+2)=0$

b. $x^2-7x+10=0$

c. $2x^2+2x-12=0$

d. $4x^2-1=0$

8-78. Sketch each parabola below with the given information.

a. A parabola with x-intercepts (2, 0) and (7, 0) and y-intercept (0, –8).

b. A parabola with exactly one x-intercept at (–1, 0) and y-intercept (0, 3).

c. The parabola represented by the equation $y=(x+5)(x-1)$.

8-79. Review the meanings of the inequality symbols in the box at right. Then decide if the statements below are true or false.

$<$ less than
$\le$ less than or equal to
$>$ greater than
$\ge$ greater than or equal to

a. $5<7$

b. $-2\ge 9$

c. $0\le 0$

d. $-5>-10$

e. $16\le -16$

f. $1>1$

8-80. Calculate the expressions below with a scientific calculator.

a. $\frac{-10+\sqrt{25}}{5}$

b. $\frac{8+\sqrt{40}}{3\cdot 3}$

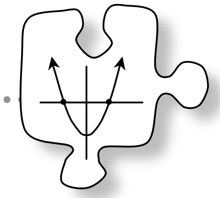
c. $\frac{8+\sqrt{3^2+2\cdot 3+1}}{-4}$

8-81. Find the equation of the line through the points (6, –8) and (0, 0).

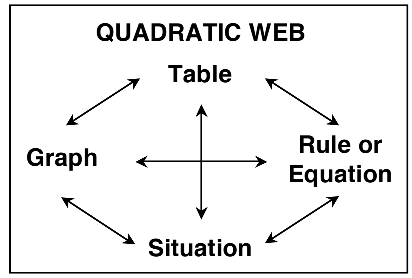
a. What is the slope of the line?

b. Is the point (3, –4) on the line? How can you tell?

8.2.5 What's the connection?



Completing the Quadratic Web



In just three lessons you have almost completed the quadratic web. Revisit the web posted in your classroom. What connections, if any, still need to be made?

Today you will focus on how to get a quadratic rule from a graph and a situation. As you work, ask yourself the following questions:

Which representation am I given?
Which representation am I looking for?

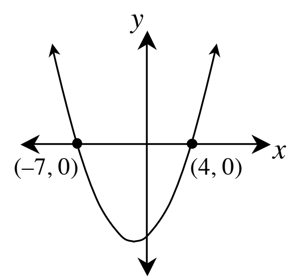
How can I **reverse** this process?

Is there another way?

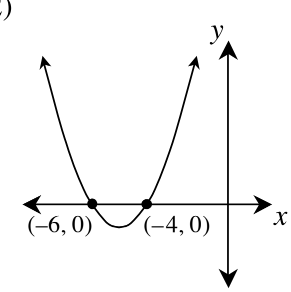
8-82. Several parabolas and quadratic rules are shown below. Match each graph with its rule. Justify your choices and share any shortcuts you find with your teammates. (Note: Not every rule will be matched with a parabola.)

Graphs:

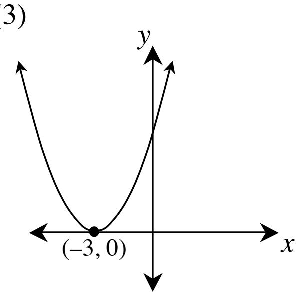
(1)



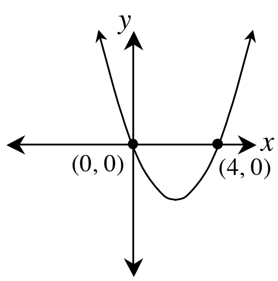
(2)



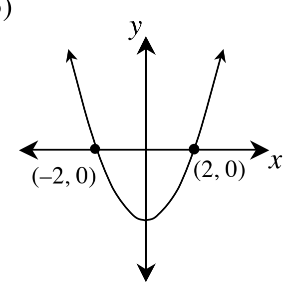
(3)



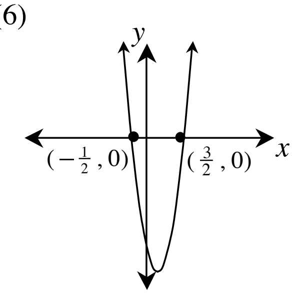
(4)



(5)



(6)



Rules:

a. $y=(x+3)^2$

b. $y=x^2+3x-28$

c. $y=x^2-11x+28$

d. $y=x^2-4$

e. $y=x^2+10x+24$

f. $y=2x^2+11x+5$

g. $y=x^2-4x$

h. $y=(x-3)^2$

i. $y=4x^2-4x-3$

8-83. QUALITY CONTROL, Part One

Congratulations! With your promotion, you are now the Quality Assurance Representative of the Function Factory. Your job is to make sure your clients are happy. Whenever a client writes to the company, you must reply with clear directions that will solve his or her problem.

Your boss has provided graphing technology and a team of fellow employees to help you fulfill your job description.

Your Task:

1. Carefully read the complaints below. Study each situation with your grapher. Work with your team to resolve each situation.
2. Write each customer a friendly response that offers a solution to his or her problem. Remember that the customers are not parabola experts! Do not assume that they know anything about parabolas.

A

Dear Ms. Quadratic,

I followed all of the directions given in your brochure on how to order a parabola. I tried to order a parabola that passed through the points (1, 0) and (–6, 0), only to have you send me the wrong one!

Please tell me how to order the correct parabola. Your immediate reply is appreciated.

Perturbed in Pennsylvania

B

Dear Ms. Quadratic,

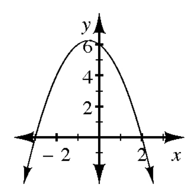
I am a very dissatisfied customer. I want a parabola that hits the x-axis only <u>once</u> at (5, 0), yet I see NO mention of this type of parabola in your pamphlet. Your company mission statement assures me that "my needs will be met no matter what." How should I order my special parabola?

Sincerely,
Troubled in Texas

C

Dear Ms. Quadratic,

Please help! I have searched through your entire brochure and did not see a parabola that would fit my needs. All I want is a parabola that looks like this:

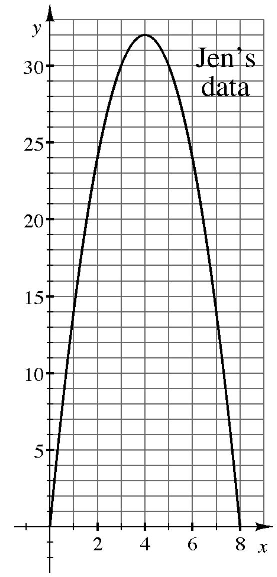


Every time I order an equation to give me this parabola you always send me a different one! I refuse to pay for any parabola but the one shown above. Please tell me how I should find the equation of this parabola or I will take my business elsewhere!

Thank you,
Agitated in Alaska

8-84. EXTRA! EXTRA!

A journalist from the school newspaper wants to publish the results from the water-balloon contest. She wants a rule for each toss so that she can program her computer to create a graph for her article. You already have rules for the tosses made by Maggie and Imp from problems 8-51 and 8-74.



a. Examine the graph at right that represents the height of Jen's toss. Find the rule for this parabola.

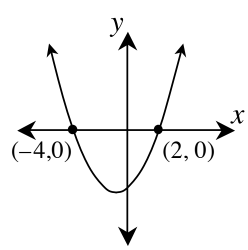
b. Al released his balloon from the 10-yard line, and it landed at the 16-yard line. If the ball reached a height of 27 yards, what equation represents the path of his toss?

Review & Preview

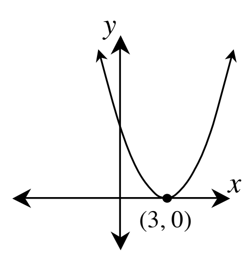
8-85. QUALITY CONTROL, Part Two

Lots O'Dough, a wealthy customer, would like to order a variety of parabolas. However, he is feeling pressed for time and said that he will pay you *lots* of extra money if you complete his order for him. Of course you agreed! He sent you sketches of each parabola that he would like to receive. Determine a possible equation for each parabola so that you can pass this information on to the Manufacturing Department.

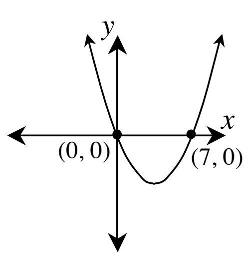
a.



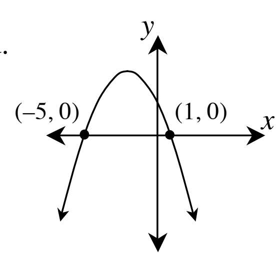
b.



c.

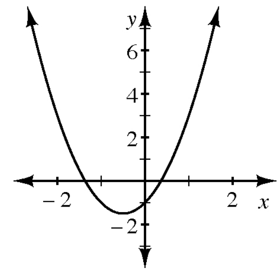


d.



8-86. Find the slope and y-intercept of the graph of $6y-3x=24$.

8-87. Examine the graph of $y=2x^2+2x-1$ at right.



a. Estimate the zeros of $2x^2+2x-1$ from the graph.

b. What happens if you try to use the Zero Product Property to find the roots of $2x^2+2x-1=0$?

8-88. Solve the equations below for x. Check your solutions.

a. $x^2+6x-40=0$

b. $2x^2+13x-24=0$

8-89. Calculate the expressions below. Then compare your answers from (a) and (b) to those in problem 8-88. What do you notice?

a. $\frac{-6+\sqrt{6^2-(4)(1)(-40)}}{2\cdot 1}$

b. $\frac{-6-\sqrt{6^2-(4)(1)(-40)}}{2\cdot 1}$

c. $\frac{-13-\sqrt{13^2-(4)(2)(-24)}}{2\cdot 2}$

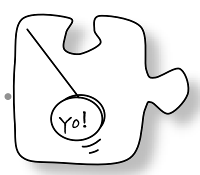
d. $\frac{-13+\sqrt{13^2-(4)(2)(-24)}}{2\cdot 2}$

8-90. Use any method to solve the systems of equations below.

a. $2x-3y=5$
$4x+y=3$

b. $m=-3+2n$
$4m+6n=-5$

8.3.1 What if it's not factorable?



Introduction to the Quadratic Formula

In Section 8.2 you developed a method to find the x-intercepts of a parabola by factoring and using the Zero Product Property. Today you will learn a new method to solve quadratic equations.

8-91. Use the Zero Product Property to find the roots of $x^2-3x-7=0$.

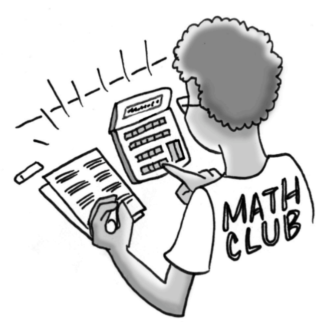
a. What happened?

b. What does this result tell you about the roots?

c. Your teacher will display the graph of $y=x^2-3x-7$ for the class. Did the graph confirm your answer to part (b)? Estimate the roots using the graph.

8-92. QUADRATIC FORMULA

Since a parabola can have x-intercepts even when its corresponding quadratic equation is not factorable, another way to find the roots of a quadratic equation is needed.



a. One way to find the roots of a quadratic equation is by using the **Quadratic Formula**, shown below. This formula uses values a, b, and c from a quadratic equation written in standard form (explained in the next paragraph).

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

When the quadratic equation is written in **standard form** (i.e., it looks like $ax^2+bx+c=0$), then a is the number of x^2-terms, b is the number of x-terms, and c is the constant. If $x^2-3x-7=0$, then what are a, b, and c?

b. The Quadratic Formula calculates *two* possible answers by using the "$\pm$" symbol. This symbol (read as "plus or minus") is shorthand notation that tells you to calculate the formula twice: once with addition and once with subtraction in the numerator. Therefore, every Quadratic Formula problem is really two different problems unless the value of $\sqrt{b^2-4ac}$ is 0.

$$\pm$$

$$x=\frac{-b+\sqrt{b^2-4ac}}{2a} \quad \text{or} \quad x=\frac{-b-\sqrt{b^2-4ac}}{2a}$$

Carefully substitute a, b, and c from $x^2-3x-7=0$ into the Quadratic Formula. Evaluate each expression (once using addition and once using subtraction) to solve for x. Do these solutions match those from part (c) of problem 8-91?

8-93. The Quadratic Formula is only one of the tools you can use to solve quadratic equations.

a. What are the other methods that you can use?

b. You may be thinking, "Where did this formula come from? Why does it work?" You can find the formula by starting with a generic quadratic $ax^2+bx+c=0$ and using your algebra skills to solve for x. See the Math Notes box for this lesson to learn about one way this formula can be derived. Later, in Chapter 12, you will learn another formal method to derive the Quadratic Formula.

8-94. Use the Quadratic Formula to solve the equations below for x, if possible. Check your solutions.

a. $3x^2+7x+2=0$

b. $2x^2-9x-35=0$

c. $8x^2+10x+3=0$

d. $x^2-5x+9=0$

8-95. In your Learning Log, describe how to use the Quadratic Formula. Be sure to include an example. Title this entry "Quadratic Formula" and include today's date.

MATH NOTES

LOOKING DEEPER

Deriving the Quadratic Formula

Why is $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ a solution of $ax^2+bx+c=0$? One way to derive this formula is shown below.

1. Begin with the quadratic equation in standard form. $$ax^2+bx+c=0$$
2. Multiply each side by $4a$. $$4a(ax^2+bx+c)=0$$ $$4a^2x^2+4abx+4ac=0$$
3. Add b^2-4ac to each side in order to get a factorable quadratic on the left. $$4a^2x^2+4abx+b^2=b^2-4ac$$
4. The left side can be factored as $(2ax+b)^2$, which is demonstrated in the generic rectangle shown at right.

b	$2abx$	b^2
$2ax$	$4a^2x^2$	$2abx$
	$2ax$	b

$$(2ax+b)^2=b^2-4ac$$

5. Take the square root of each side. Since both the positive and negative values of a number can be squared to give the same result (for example, 4^2 and $(-4)^2$ both equal 16), then there are two possible square root values: $\sqrt{b^2-4ac}$ and $-\sqrt{b^2-4ac}$. $$2ax+b=\pm\sqrt{b^2-4ac}$$
6. Now continue to solve for x by subtracting b from both sides and dividing by $2a$. Notice that a cannot equal zero or else you will get an error! However, if $a=0$, then this equation would not be quadratic and you would not use this formula. $$2ax=-b\pm\sqrt{b^2-4ac}$$ $$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
7. Thus, $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ are solutions of the equation $ax^2+bx+c=0$.

Review & Preview

8-96. Solve the following quadratic equations by factoring and using the Zero Product Property. Be sure to check your solutions.

a. $x^2 - 13x + 42 = 0$

b. $0 = 3x^2 + 10x - 8$

c. $2x^2 - 10x = 0$

d. $4x^2 + 8x - 60 = 0$

8-97. Use the Quadratic Formula to solve $x^2 - 13x + 42 = 0$. Did your solution match the solution from part (a) of problem 8-96?

8-98. Does a quadratic equation always have two solutions? That is, does a parabola always intersect the x-axis twice?

a. If possible, draw an example of a parabola that only intersects the x-axis once.

b. What does it mean if the quadratic equation has no solution? Draw a possible parabola that would cause this to happen.

8-99. Find the equation of the line through the point (–2, 8) with slope $\frac{1}{2}$.

8-100. For each of the following equations, indicate whether its graph would be a line or a parabola.

a. $5x + 2y = 7$

b. $y = 3x^2$

c. $y = 3$

d. $4x^2 + 3x = 7 + y$

8-101. **Multiple Choice:** Which equations below are equivalent to:

$$\frac{1}{2}(6x - 14) + 5x = 2 - 3x + 8 \text{ ?}$$

a. $3x - 7 + 5x = 10 - 3x$

b. $3x - 14 + 5x = 2 - 3x + 8$

c. $8x - 14 = 10 - 3x$

d. $6x - 14 + 10x = 4 - 6x + 16$

8-102. Review the descriptions for the inequality symbols $<$, $\le$, $>$, and $\ge$ in problem 8-79. Then decide if the statements below are true or false.

a. $11 < -13$

b. $5 \cdot 2 \ge 10$

c. $13 > -3(2 - 6)$

d. $4 \le 4$

e. $9 \ge -9$

f. $-2 > -2$

g. $-16 < -15$

h. $0 > 6$

8.3.2 What if the equation is not in standard form?

More Solving Quadratic Equations

Today you will **apply** and **extend** what you know about solving quadratic equations.

8-103. For the quadratic equation $6x^2 + 11x - 10 = 0$:

a. Solve it using the Zero Product Property.

b. Solve it using the Quadratic Formula.

c. Did the solutions from parts (a) and (b) match? If not, why not?

8-104. As the Math Notes box from Lesson 8.3.1 demonstrated, the Quadratic Formula can solve any quadratic equation $ax^2 + bx + c = 0$ if $a \neq 0$. But what if the equation is not in standard form? What if terms are missing? Consider these questions as you solve the quadratic equations below. Share your ideas with your teammates and be prepared to demonstrate your process for the class.

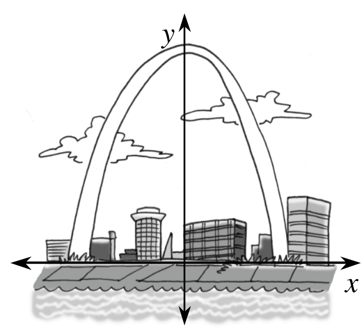
a. $4x^2 - 121 = 0$

b. $2x^2 - 2 - 3x = 0$

c. $15x^2 - 165x = 630$

d. $36x^2 + 25 = 60x$

8-105. THE SAINT LOUIS GATEWAY ARCH



The Saint Louis Gateway Arch (pictured at right) has a shape much like a parabola. Suppose the Gateway Arch can be approximated by $y = 630 - 0.00635x^2$, where both x and y represent distances in feet and the origin is the point on the ground directly below the arch's apex (its highest point).

a. Find the x-intercepts of the Gateway Arch. What does this information tell you? Use a calculator to evaluate your answers.

b. How wide is the arch at its base?

c. How tall is the arch? How did you find your solution?

d. Draw a quick sketch of the arch on graph paper, labeling the axes with all of the values you know.

MATH NOTES

METHODS AND MEANINGS

Solving a Quadratic Equation

So far in this course, you have learned two algebraic methods to solve a quadratic equation of the form $ax^2 + bx + c = 0$.

One of these methods, the Zero Product Property, requires the equation to be a product of factors that equal zero. In this case, the quadratic equation must be factored, as shown in Example 1 below. Another strategy uses the Quadratic Formula, as demonstrated in Example 2 below. Notice that each strategy results in the same answer.

Example 1: Solve $3x^2 + x - 14 = 0$ for x using the Zero Product Property.

Solution: First, factor the quadratic so it is written as a product: $(3x+7)(x-2) = 0$. The Zero Product Property states that if the product of two terms is 0, then at least one of the factors must be 0. Thus, $3x + 7 = 0$ or $x - 2 = 0$.

Solving these equations for x reveals that $x = -\frac{7}{3}$ or that $x = 2$.

Example 2: Solve $3x^2 + x - 14 = 0$ for x using the Quadratic Formula.

Solution: First, identify a, b, and c. a equals the number of x^2-terms, b equals the number of x terms, and c equals the constant. For $3x^2 + x - 14 = 0$, $a = 3$, $b = 1$, and $c = -14$. Substitute the values of a, b, and c into the Quadratic Formula and evaluate the expression twice: once with addition and once with subtraction. Examine this method below:

$$\begin{aligned} x &= \frac{-1+\sqrt{1^2-4(3)(-14)}}{2\cdot 3} \\ &= \frac{-1+\sqrt{169}}{6} \\ &= \frac{-1+13}{6} \\ &= \frac{12}{6} \\ &= 2 \end{aligned}$$

or

$$\begin{aligned} x &= \frac{-1-\sqrt{1^2-4(3)(-14)}}{2\cdot 3} \\ &= \frac{-1-\sqrt{169}}{6} \\ &= \frac{-1-13}{6} \\ &= \frac{-14}{6} \\ &= -\frac{7}{3} \end{aligned}$$

Review & Preview

8-106. Solve the following quadratic equations by factoring and using the Zero Product Property. Then check your solutions.

a. $x^2 - 10x + 25 = 0$

b. $0 = 3x^2 + 17x - 6$

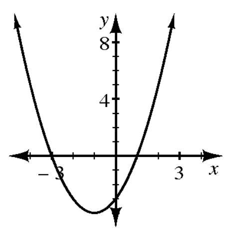
c. $3x^2 - 2x = 5$

d. $16x^2 - 9 = 0$

8-107. Use the Quadratic Formula to solve part (b) of problem 8-106 above. Did your solution match the solution you got by factoring and using the Zero Product Property (in part (b) of problem 8-106)?

8-108. Find the equation of each parabola below based on the given information.

a.



b.

x	–4	–3	–2	–1	0	1	2	3	4
y	12	5	0	–3	–4	–3	0	5	12

8-109. Solve the following problem using any method. Write your solution as a sentence.

The length of a rectangle is 5 cm longer than twice the length of the width. If the area of the rectangle is 403 square centimeters, what is the width?

8-110. Which of the points below is a solution to $4x - 3y = 10$? Note: More than one point may make this equation true.

a. (1, 2)

b. (4, 2)

c. (7, 6)

d. (4, –3)

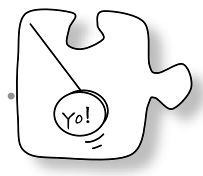
8-111. Kristen loves shortcuts. She figured out that she can find *x*- and *y*-intercepts for any line without graphing! For example, she knows that the *x*-intercept for $5x - 3y = 15$ is (3, 0) just by examining the rule.

a. What is her shortcut?

b. Does this shortcut work for the *y*-intercept? Try it and then test your result by changing $5x - 3y = 15$ into $y = mx + b$ form.

c. Use this shortcut to find the *x*- and *y*-intercepts of $3x - 2y = 24$.

8.3.3 Which method should I use?



Choosing a Strategy

You now have two algebraic methods to solve quadratic equations: using the Zero Product Property and using the Quadratic Formula. How can you decide which strategy is best to try first? By the end of this lesson, you should have some strategies to help you determine which method to try first when solving a quadratic equation.

8-112. Examine the quadratic equations below with your team. For each equation:

- Decide which strategy is best to try first.
- Solve the equation. If your first strategy does not work, switch to the other strategy.
- Check your solution(s).

Be prepared to share your process with the class.

a. $x^2 + 12x + 27 = 0$

b. $0.5x^2 + 9x + 3.2 = 0$

c. $(3x + 4)(2x - 1) = 0$

d. $x^2 + 16 = 8x$

e. $x^2 + 5 - 2x = 0$

f. $20x^2 - 30x = 2x + 45$

8-113. With the class, decide when it is best to solve a quadratic by factoring and when you should go directly to the Quadratic Formula. Copy your observations in your Learning Log. Title this entry “Choosing a Strategy to Solve Quadratics” and include today’s date.

8-114. While solving $(x-5)(x+2)=-6$, Kyle decided that x must equal 5 or –2. "Not so fast!" exclaimed Stanton, "The product does not equal zero. We need to change the equation first."

a. What is Stanton talking about?

b. How can the equation be rewritten? Discuss this with your team and use your algebraic tools to rewrite the equation so that it can be solved.

c. Solve the resulting equation from part (b) for x. Do your solutions match Kyle's?

8-115. MOE'S YO

Moe is playing with a yo-yo. He throws the yo-yo down and then pulls it back up. The motion of the yo-yo is represented by the equation $y=2x^2-4.8x$, where x represents the number of seconds since the yo-yo left Moe's hand, and y represents the vertical height in inches of the yo-yo with respect to Moe's hand. Note that when the yo-yo is in Moe's hand, $y = 0$, and when the yo-yo is below his hand, y is negative.

a. How long is Moe's yo-yo in the air before it comes back to Moe's hand? Write and solve a quadratic equation to find the times that the yo-yo is in Moe's hand.

b. At what time does the yo-yo turn around? Use what you know about parabolas to help you.

c. How long is the yo-yo's string? That is, what is y when the yo-yo changes direction?

d. Draw a sketch of the graph representing the motion of Moe's yo-yo. On the sketch, label the important points: when the yo-yo is in Moe's hand and when it changes direction.

MATH NOTES

LOOKING DEEPER

Simplifying Square Roots

Before calculators were universally available, people who wanted to use approximate decimal values for numbers like $\sqrt{45}$ had a few options:

1. Carry around copies of long square-root tables.
2. Use Guess and Check repeatedly to get desired accuracy.
3. "Simplify" the square roots. A square root is **simplified** when there are no more perfect square factors (square numbers such as 4, 25, and 81) under the radical sign.

Simplifying square roots was by far the fastest method. People factored the number as the product of integers hoping to find at least one perfect square number. They memorized approximations of the square roots of the integers from one to ten. Then they could figure out the decimal value by multiplying these memorized facts with the roots of the square numbers. Here are some examples of this method.

Example 1: Simplify $\sqrt{45}$.

First rewrite $\sqrt{45}$ in an equivalent factored form so that one of the factors is a perfect square. Simplify the square root of the perfect square. Verify with your calculator that both $3\sqrt{5}$ and $\sqrt{45} \approx 6.71$.

Examine **Example 2** and **Example 3** at right. Note that in Example 3, $\sqrt{72}$ was rewritten as $\sqrt{36} \cdot \sqrt{2}$, rather than as $\sqrt{9} \cdot \sqrt{8}$ or $\sqrt{4} \cdot \sqrt{18}$, because 36 is the largest perfect square factor of 72. However, since

$$\sqrt{4} \cdot \sqrt{18} = 2\sqrt{9 \cdot 2} = 2\sqrt{9} \cdot \sqrt{2} = 2 \cdot 3\sqrt{2} = 6\sqrt{2} \quad \text{and}$$
$$\sqrt{9} \cdot \sqrt{8} = 3\sqrt{4 \cdot 2} = 3\sqrt{4} \cdot \sqrt{2} = 3 \cdot 2\sqrt{2} = 6\sqrt{2},$$

you can still get the same answer if you simplify it using different methods.

Example 1

$$\sqrt{45} = \sqrt{9 \cdot 5}$$
$$= \sqrt{9} \cdot \sqrt{5}$$
$$= 3\sqrt{5}$$

Example 2

$$\sqrt{27}$$
$$= \sqrt{9} \cdot \sqrt{3}$$
$$= 3\sqrt{3}$$

Example 3

$$\sqrt{72}$$
$$= \sqrt{36} \cdot \sqrt{2}$$
$$= 6\sqrt{2}$$

When you take the square root of an integer that is not a perfect square, the result is a decimal that never repeats itself and never ends. This result is called an **irrational number**. The irrational numbers and the rational numbers together form the numbers we use in this course, which are called **real numbers**.

Generally, since it is now the age of technology, when a decimal approximation of an irrational square root is desired, a calculator is used. However, for an exact answer, the number must be written using the $\sqrt{}$ symbol.

Review & Preview

8-116. Write and solve an equation (or system of equations) for the situation described below. Define your variable(s) and write your solution as a sentence.

Daria has 18 coins that are all nickels and quarters. The number of nickels is 3 more than twice the number of quarters. If she has $1.90 in all, how many nickels does Daria have?

8-117. Solve the following quadratic equations using any method.

a. $10000x^2 - 64 = 0$

b. $9x^2 - 8 = -34x$

c. $2x^2 - 4x + 7 = 0$

d. $3.2x + 0.2x^2 - 5 = 0$

8-118. Find a rule that represents the number of tiles in Figure *x* for the tile pattern at right.

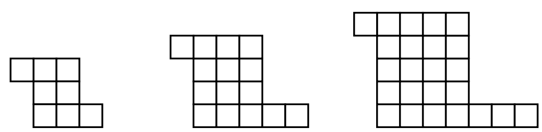


Figure 1 Figure 2 Figure 3

8-119. Solve the equations below for *x*. Check your solutions.

a. $3x^2 + 3x = 6 + 3x^2$

b. $\frac{5}{x} = \frac{1}{3}$

c. $5 - (2x - 3) = -3x + 6$

d. $6(x - 3) + 2x = 4(2x + 1) - 22$

8-120. Line *L* passes through the points (–44, 42) and (–31, 94), while line *M* has the rule $y = 6 + 3x$. Which line is steeper? **Justify** your answer.

8-121. **Multiple Choice:** Which line below is perpendicular to the line $2x - 5y = 3$?

a. $2x + 5y = 7$

b. $-2x + 5y = 4$

c. $5x - 2y = -1$

d. $5x + 2y = 3$

Chapter 8 Closure What have I learned?

Reflection and Synthesis

The activities below offer you a chance to reflect on what you have learned during this chapter. As you work, look for concepts that you feel very comfortable with, ideas that you would like to learn more about, and topics you need more help with. Look for **connections** between ideas as well as **connections** with material you learned previously.

① TEAM BRAINSTORM

With your team, brainstorm a list for each of the following topics. Be as detailed as you can. How long can you make your list? Challenge yourselves. Be prepared to share your team's ideas with the class.

Topics: What have you studied in this chapter? What ideas and words were important in what you learned? Remember to be as detailed as you can.

Ways of Thinking: What Ways of Thinking did you use in this chapter? When did you use them?

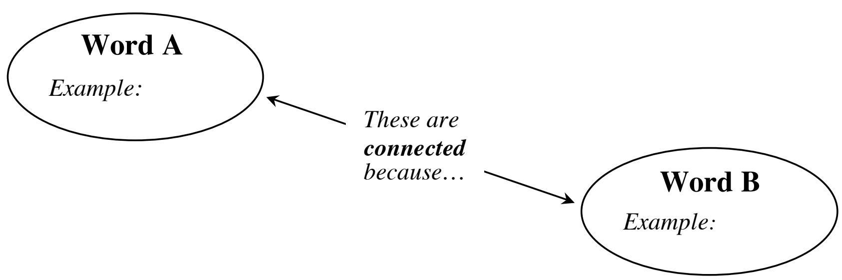
Connections: What topics, ideas, and words that you learned *before* this chapter are **connected** to the new ideas in this chapter? Again, make your list as long as you can.

② MAKING CONNECTIONS

The following is a list of the vocabulary used in this chapter. The words that appear in bold are new to this chapter. Make sure that you are familiar with all of these words and know what they mean. Refer to the glossary or index for any words that you do not yet understand.

binomial	**factor**	generic rectangle
graph	**monomial**	parabola
product	**quadratic equation**	**Quadratic Formula**
root	solution	**standard form for quadratics**
sum	**symmetry**	**trinomial**
vertex	x-intercept	$x \rightarrow y$ table
y-intercept	**Zero Product Property**	

Make a concept map showing all of the **connections** you can find among the key words and ideas listed above. To show a **connection** between two words, draw a line between them and explain the **connection**, as shown in the example below. A word can be **connected** to any other word as long as there is a **justified connection**. For each key word or idea, provide a sketch that illustrates the idea (see the example below).



Your teacher may provide you with vocabulary cards to help you get started. If you use the cards to plan your concept map, be sure either to re-draw your concept map on your paper or to glue the vocabulary cards to a poster with all of the **connections** explained for others to see and understand.

While you are making your map, your team may think of related words or ideas that are not listed above. Be sure to include these ideas on your concept map.

③ SUMMARIZING MY UNDERSTANDING

This section gives you an opportunity to show what you know about certain math topics or ideas. Your teacher will give you directions for exactly how to do this. Your teacher may also provide a "GO" page to work on. The "GO" stands for "Graphic Organizer," a tool you can use to organize your thoughts and communicate your ideas clearly.

④ WHAT HAVE I LEARNED?

This section will help you evaluate which types of problems you have seen with which you feel comfortable and those with which you need more help. Even if your teacher does not assign this section, it is a good idea to try these problems and find out for yourself what you know and what you need to work on.

Solve each problem as completely as you can. The table at the end of the closure section has answers to these problems. It also tells you where you can find additional help and practice on problems like these.

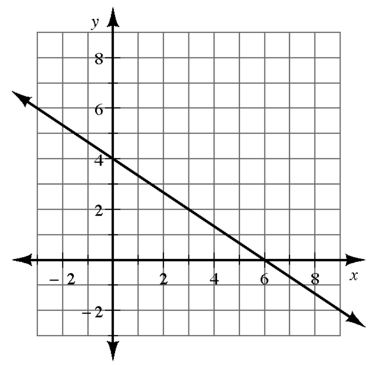
CL 8-122. For the graph of the line at right:

a. Find the slope.

b. Find the y-intercept.

c. Find an equation.

d. Find an equation of a line perpendicular to this one that passes through $(0,7)$.



CL 8-123. Factor and use the Zero Product Property to find the roots of the following quadratic equations.

a. $0 = x^2 - 7x + 12$

b. $0 = 6x^2 - 23x + 20$

c. $0 = x^2 - 9$

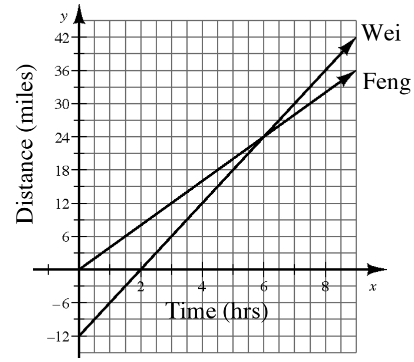
d. $0 = x^2 + 12x + 36$

CL 8-124. Use the Quadratic Formula to solve these equations.

a. $0 = x^2 - 7x + 3$

b. $3x^2 + 5x + 1 = 0$

CL 8-125. Use the graph at right to answer the questions below.



a. One of these lines represents Feng, and one represents Wai. Write an equation for each girl's line.

b. The two girls are riding bikes. How fast does each girl ride?

c. When do Feng and Wai meet? At that point, how far are they from school?

CL 8-126. Factor each expression below completely.

a. $3x^2 + 21x + 30$

b. $7x^2 - 63$

CL 8-127. Find the coordinates of the y-intercept, x-intercepts, and vertex of $y = x^2 - 2x - 15$. Show all of the work that you do to find these points.

CL 8-128. Solve for x using the method of your choice.

a. $0 = 2x^2 - 5x - 33$

b. $0 = 3x^2 - 4x - 1$

CL 8-129. Quinn started off with twice as much candy as Denali, but then he ate 4 pieces. When Quinn and Denali put their candy together, they now have a total of 50 pieces. How many pieces of candy did Denali start with?

CL 8-130. Check your answers using the table at the end of the closure section. Which problems do you feel confident about? Which problems were hard? Use the table to make a list of topics you need help on and a list of topics you need to practice more.

⑤ HOW AM I THINKING?

This course focuses on five different **Ways of Thinking**: reversing thinking, justifying, generalizing, making connections, and applying and extending understanding. These are some of the ways in which you think while trying to make sense of a concept or to solve a problem (even outside of math class). During this chapter, you have probably used each Way of Thinking multiple times without even realizing it!

Choose three of these Ways of Thinking that you remember using while working in this chapter. For each Way of Thinking that you choose, show and explain where you used it and how you used it. Describe why thinking in this way helped you solve a particular problem or understand something new. (For instance, explain why you wanted to **generalize** in this particular case, or why it was useful to see these particular **connections**.) Be sure to include examples to demonstrate your thinking.

Answers and Support for Closure Activity #4
What Have I Learned?

Problem	Solution	Need Help?	More Practice
CL 8-122.	a. $m=-\frac{2}{3}$ b. y-intercept: 4 c. $y=-\frac{2}{3}x+4$ d. $y=\frac{3}{2}x+7$	Lessons 7.1.3, 7.1.5, 7.2.2, 7.2.3, and 7.3.2 Math Notes boxes	Problems 8-9, 8-31, 8-57, 8-86, 8-99, and 8-121
CL 8-123.	a. $x = 4$ or $x = 3$ b. $x = \frac{5}{2}$ or $x = \frac{4}{3}$ c. $x = -3$ or $x = 3$ d. $x = -6$	Lessons 8.1.4, 8.2.3, and 8.3.2 Math Notes boxes	Problems 8-66, 8-72, 8-77, 8-88, 8-96, and 8-106
CL 8-124.	a. $x=\frac{7\pm\sqrt{37}}{2}$ ($x \approx 6.54$ or 0.46) b. $x=\frac{-5\pm\sqrt{13}}{6}$ ($x \approx -0.23$ or -1.43)	Problem 8-92, Lessons 8.3.1 and 8.3.2 Math Notes boxes	Problems 8-94, 8-97, 8-104, and 8-107

Problem	Solution	Need Help?	More Practice
CL 8-125.	a. Feng: $y = 4x$ Wai: $y = 6x - 12$ b. Feng rides at 4 miles per hour; Wai rides at 6 miles per hour. c. Feng and Wai meet after 6 hours. At that point, they are 24 miles from school.	Lessons 7.1.5 and 7.2.2 Math Notes boxes	See Lessons 7.2.2 and 7.2.3
CL 8-126.	a. $3(x+2)(x+5)$ b. $7(x+3)(x-3)$	Problems 8-13, 8-14, and 8-35; Lesson 8.1.5 Math Notes box	Problems 8-15, 8-16, 8-22, 8-23, 8-24, 8-29, 8-33, 8-36, 8-37, and 8-69
CL 8-127.	y-intercept: -15 x-intercepts: 5 and -3 vertex: $(1, -16)$	Problems 8-43, 8-51, 8-59, 8-60, 8-61, and 8-62	Problems 8-10, 8-54, 8-68, and 8-82
CL 8-128.	a. $x = \frac{11}{2}$ or $x = -3$ b. $x = \frac{4 \pm \sqrt{28}}{6} = \frac{4 \pm 2\sqrt{7}}{6} = \frac{2 \pm \sqrt{7}}{3}$ ($x \approx 1.55$ or $x \approx -0.22$)	Lessons 8.1.4, 8.2.3, 8.3.1, 8.3.2	Problems 8-112 and 8-117
CL 8-129.	Denali has 18 pieces of candy.	Lesson 7.1.4 Math Notes box	Problem 8-116