

INEQUALITIES

9



CHAPTER 9

Inequalities

So far in this course you have focused on what you can determine when two expressions are equal. By using what you know about balancing equations, you can now solve linear and quadratic equations for a given variable.

However, what if the two expressions are not equal? If you know that one expression is always larger than the other, what does that tell you about the variable? In this chapter you will learn how to deal with these types of relationships, called *inequalities*, and will develop ways to represent solutions to inequalities both algebraically and graphically.

In addition, you will extend your ability to work with mathematical sentences by learning how to write inequalities from word problems.

In this chapter, you will learn:

- How to write an inequality to represent a word problem.
- How to solve linear inequalities and represent the solutions on a number line.
- How to represent the solutions of linear and nonlinear inequalities with two variables on a graph.
- How to graph a system of inequalities.

Guiding Questions

Think about these questions throughout this chapter:

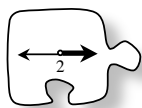
How can I represent it algebraically?

How can I solve it?

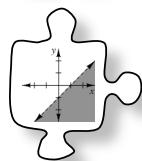
What is a solution?

What is the connection?

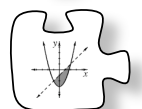
Chapter Outline



Section 9.1 In this section, you will study how to solve linear inequalities and apply this understanding to solving applications.



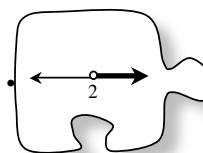
Section 9.2 After learning how to represent solutions to one-variable inequalities on a number line, you will study how to represent the solutions of two-variable inequalities on an $x \rightarrow y$ graph.



Section 9.3 In the final section, you will apply what you know about systems of equations to help find the solutions to a system of inequalities.

9.1.1 What if the quantities are not equal?

Solving Linear, One-Variable Inequalities



In this course, you have developed a variety of skills to find solutions to different kinds of equations. Now you will **apply** these equation-solving skills to solve inequalities.



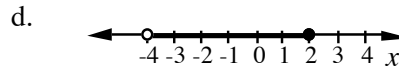
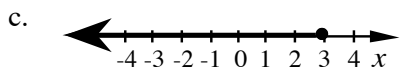
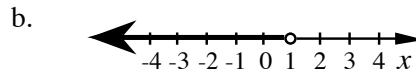
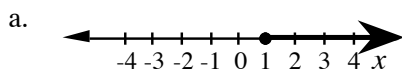
9-1. As a class, create a “human number line” for each of the following mathematical sentences. You will be assigned a number to represent on the number line. When your number makes the equation or inequality true, stand up to show that your number is a solution. If your number does not make the equation or inequality true, remain seated.

- | | | | |
|----------------|-----------------------|-----------------|---------------|
| a. $x \geq -2$ | b. $x \leq 1$ | c. $x = 3$ | d. $x \geq 0$ |
| e. $x = -2$ | f. $-1 \leq x \leq 4$ | g. $x^2 \geq 4$ | h. $x < -3$ |

9-2. Based on your observations from problem 9-1, discuss the following questions with your class. Be sure to **justify** your responses.

- Compare the solutions to an inequality (like $x \geq -2$) with that of an equation (like $x = 3$). What is different? What causes this to happen?
- How many solutions does an inequality such as $x \leq 1$ have?
- How is the result of $-1 \leq x \leq 4$ different from the other inequalities? What about the result of $x^2 \geq 4$?

9-3. Write an inequality that represents the solutions on each number line below.



9-4. SOLUTIONS TO A LINEAR INEQUALITY

With your study team, find at least five x -values that make the inequality below true:

$$2x - 5 \geq 3$$

- How many solutions are there?
- What is the smallest solution for x ? This point is called a **boundary point**.
- What is the significance of the boundary point? What is its relationship with the inequality $2x - 5 \geq 3$?
- Write an inequality that represents the solutions for x . On a number line, highlight the solutions for x . Be ready to share your number line with the class.

9-5. SOLVING LINEAR INEQUALITIES WITH ONE VARIABLE

Analyze the process for solving an inequality, such as $3 - 2x < 1$, by addressing the questions below.

- The key point to start with is the **boundary point**. How can you quickly solve for this point? Once you have determined your strategy, find the boundary point for $3 - 2x < 1$.
- Decide if the boundary point is part of the solution to the inequality. If it is part of the solution, indicate this on a number line with a solid point. If it is not a solution, show this by using an **unfilled circle** as a boundary.
- Finally, to determine on which side of the boundary the solutions lie, choose a point to test in the inequality. If the point is a solution, then all points on that side of the boundary are part of the solution. If the point is not a solution, what does that tell you about the solutions? Write your solutions to $3 - 2x < 1$ as an inequality and represent the solutions on a number line.

9-6. With your study team, find all of the solutions to the inequality $3x + 1 < 7$. Decide how to represent these solutions on a number line and be prepared to **justify** your decisions to the class.



METHODS AND MEANINGS

Just as the symbol “=” is used to represent that two quantities are equal in mathematics, the **inequality symbols** at right are used when describing the relationships between quantities that are not necessarily equal.

When graphing an inequality on a number line, such as $x \geq 4$, a solid point indicates that the value is a solution of the inequality. However, an unfilled circle indicates that the value is not part of the solution.

Inequality Symbols

- < less than
- ≤ less than or equal to
- > greater than
- ≥ greater than or equal to



- 9-7. Solve the problem below by writing and solving an equation. A Guess and Check table may help you write the equation. Be sure to define your variable.

There are a total of 122 countries in Africa, Europe, and North America (as of 2003). Europe has twice as many countries as North America, and Africa has seven more than Europe. How many countries are in each of these three continents? Write an equation and solve it to answer this question.



- 9-8. Solve each of the following inequalities for the given variable. Represent your solutions on a number line.

a. $2(3p + 1) > -4$

b. $9k - 2 < 3k + 10$

c. $5 - h \geq 4$

9-9. Solve the following quadratic equations. Check your solutions, if possible.

a. $2k^2 + k - 6 = 0$

b. $m^2 = 9$

c. $w(2w + 8) = 24$

d. $3n^2 - 4n = 5$

9-10. Identify the statements below as sometimes true, always true, or never true.

$<$	less than
\leq	less than or equal to
$>$	greater than
\geq	greater than or equal to

a. $-4 \leq 9$

b. $x < 1$

c. $-5 > -2$

d. $3x + 5 = 2$

e. $61 = 61$

f. $-6 < -6$

9-11. Assuming that x does not equal zero, what is $\frac{x}{x}$? Explain how you know.

9-12. Robbie builds model rockets. One day he sets up a rocket, backs away from the launch pad, and then shoots the rocket off into the air. The rocket's path is represented by the equation $y = -10x^2 + 130x - 400$, where y is the height in meters off the ground and x is the horizontal distance in meters from Robbie.



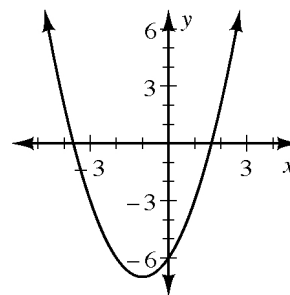
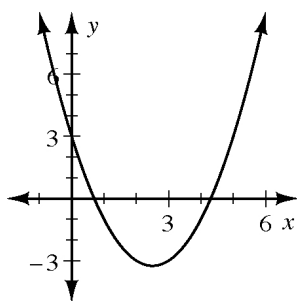
a. Use either the Zero Product Property or the Quadratic Formula to find the x -intercepts of the path of Robbie's rocket. What do the x -intercepts tell you?

b. When Robbie's rocket lands, how far is it from the launch pad?

9-13. For each parabola graphed below, visually estimate the x -intercepts. Then use the Quadratic Formula to confirm your estimates.

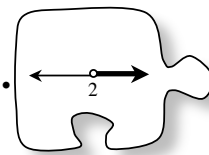
a. $y = x^2 - 5x + 3$

b. $y = x^2 + 2x - 6$



9.1.2 How can I use inequalities?

More Solving Inequalities



In Lesson 9.1.1 you learned how to solve inequalities with one variable. Today you will focus on special inequalities and learn how you can use inequalities to solve an application problem.

9-14. Review what you learned in Lesson 9.1.1 to solve the inequalities below. Represent your solutions both as an inequality and on a number line.

- a. $x - 7 < -2$ b. $3m + 2 \leq 8m - 8$ c. $\frac{2}{3}p - 2 > -4$
d. $2 - 3(x - 1) \geq x - 7$ e. $9k - 4 + 1 \leq 2k - 3 + 7k$ f. $3y + 1 < 3y + 1$

9-15. THE UNITED NATIONS

At the end of this chapter, your team will have the exciting responsibility of representing a country at a special meeting of the United Nations (U.N.). The U.N. needs your help preparing for future large-scale disasters. You will need to help find a solution that not only works best for the country you represent, but that also accommodates the needs of each of the other countries. To prepare you for this task, this chapter will present daily problems to familiarize you with the important issues and concerns of other countries.



Start by writing and solving an equation (or system of equations) that represents the problem below. Be sure to define any variables you use.

Turkey has a population of 66 million people and is made up almost entirely of two ethnic groups: Turks and Kurds. There are four times more Turks than Kurds. Write an equation and solve it to find out how many Kurds live in Turkey.

- 9-16. In 1912, Japan gave the United States several thousand flowering cherry trees as a symbol of friendship. Similarly, the nation of Cameroon plans to give flowering Satta trees to other countries this year. When asked how to decide which Satta trees make good gifts, Cameroon's chief arborist explained:

"We plant Satta trees when they are 6 cm tall, and they grow 9 cm every year. The trees only flower when they are taller than 150 cm."



It is very important that the trees Cameroon gives flower this year! It would be considered an insult to receive a tree that did not bloom. Luckily, Cameroon has many groves of Satta trees from which to select its gifts. How old must the trees be so that they will flower within the year?

- Discuss with your study team whether an inequality or an equation is appropriate for this situation. Be prepared to share your reasoning.
- Write and solve a mathematical sentence to determine how old the trees can be so that they flower this year.
- Later, the arborist added:

"I almost forgot to tell you! When the trees become very old, they stop flowering. Make sure you choose trees that are no more than 240 cm tall!"

Discuss with your team how you can use this additional information to make sure you choose trees that will flower. Be prepared to share your answer with the class.



9-17. Solve the inequalities below for the given variables. Represent your solutions on a number line.

a. $3(2k - 1) < 9$

b. $\frac{2p}{5} \leq 6$

c. $-2 + 8n > 2$

d. $7t - 4 \geq 2t - 4$

9-18. Use your graphing shortcuts to graph $y = -2x + 3$. Identify the x - and y -intercepts.

9-19. Find the equation of the line with slope $-\frac{3}{5}$ passing through the point $(-6, 2)$.

9-20. Use a generic rectangle to multiply $(x + 2)(3x - 5)$.

a. What is $(3x^2 + x - 10) \div (x + 2)$? How do you know?

b. Likewise, determine $(3x^2 + x - 10) \div (3x - 5)$.

9-21. Solve the quadratic equation below. Check your solutions with a calculator.

$$3x^2 + 2.5x = 12.5$$

9-22. Factor the expressions below completely, if possible.

a. $4x^2 - 20x + 25$

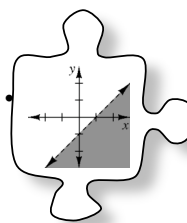
b. $x^2 + 11x - 2$

c. $3x^2 - 12x$

d. $10x^2 - 35x - 20$

9.2.1 What if the inequality has two variables?

Graphing Two-Variable Inequalities



In Section 9.1, you learned how to use an inequality with one variable to help solve a word problem. You also discovered that a one-variable inequality can have zero, one, or more solutions and that these solutions can be represented on a number line. But what if an inequality has two variables? What is a solution to a two-variable inequality? And how could these solutions be represented graphically?

9-23. EXAMINING THE SOLUTIONS OF A LINEAR EQUATION

Find your graph of $y = -2x + 3$ from problem 9-18. Compare your graph with the poster graph provided by your teacher.

- Is the point $(-1, 5)$ a solution to the equation $y = -2x + 3$? How can you tell by looking at the graph? How can you tell by using the equation?
- Is the point $(2, -1)$ a solution? What about the point $(0, 0)$? **Justify** each conclusion with both the graph and the equation.
- What determines if a point lies on the line? What is the difference between the points on the line and the points not on the line?

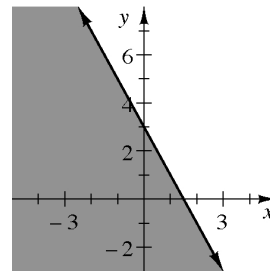
9-24. GRAPHING A LINEAR INEQUALITY

In problem 9-23, you found that the points on the line are the *only* points that make the equation $y = -2x + 3$ true. But what if you want to graph the solutions for the inequality $y \geq -2x + 3$? How will that graph differ from the graph of $y = -2x + 3$? Consider this question as you follow the steps below.

- Your team will be given a list of points to test in the inequality $y \geq -2x + 3$. For each point that makes the inequality true, place a sticky dot on that point on the class graph.
- Now examine the solutions shown on the graph. With your team, discuss the questions below. Be ready to share your discoveries with the class.
 - Are there any points on the graph that you suspect are solutions but do not have a sticker?
 - Are there any stickers that you think may be misplaced? If so, verify these points so that you can have a complete graph of the solutions.
 - What about the points on the line? Are they all solutions to the inequality $y \geq -2x + 3$? Why or why not?
 - How many solutions are there?
 - Why aren't any of the solutions located below the line?

9-25. What else can you learn about solutions of linear inequalities? Think about this as you answer the questions below with your team.

a. What if the graph were shaded like the one at right? What inequality would correspond with this graph?



b. Heidi asks, “What if I changed the inequality to be $y < -2x + 3$? Now what would the graph look like?” Discuss this with your teammates and decide the best way to represent the solutions to the inequality $y < -2x + 3$. Be prepared to share your graph with the class.

9-26. Graph the inequalities below on graph paper. For each inequality:

- Graph the boundary as either a solid or a dashed line.
- Shade the region that makes the inequality true.

a. $y > -\frac{1}{3}x - 1$

b. $y \leq 4x + 2$

c. $y < \frac{5}{2}x + 3$

d. $2x - y \leq 5$

9-27. In your Learning Log, explain how to graph a linear inequality. Be sure to address the questions below. Title this entry “Graphing Linear Inequalities” and include today’s date.



- How can you determine if the line is part of the solution?
- How can you determine which side of the line the solution belongs to?
- What point(s) is (are) easiest to test?
- How many points do you need to test?


Review & Preview

- 9-28. Represent the solutions to the inequalities below on a number line.
- a. $3x - 2 < 10$
 - b. $5x - 1 - 3x \geq 4x + 5$
 - c. $2(x + 2) > 10 - x$
 - d. $4(x - 3) + 5 \geq -7$

- 9-29. Algeria has decided to take out an advertisement in the U.N newspaper, *Liberty Daily*. The newspaper charges a base fee of \$1200 for an ad. There is an additional fee of \$300 for every inch in height. If Algeria is willing to spend any amount up to (and including) \$2700, what choices does the country have for the height of the ad?



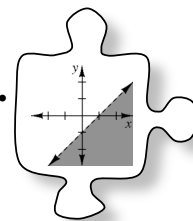
- 9-30. Solve the problem below by writing and solving one or two equations. A Guess and Check table may help you get started. Be sure to define your variable(s) and write your solution as a sentence.

Rowan received 3 points for each question he answered correctly on Part 1 of a test and 2 points for each question he answered correctly on Part 2. If he answered 33 questions correctly and received a total of 85 points, how many questions did he answer correctly on Part 1?

- 9-31. Line m has intercepts $(-7, 0)$ and $(0, -2)$.
- a. Find the equation of line m .
 - b. Is the point $(49, -16)$ also on line m ? How do you know?
 - c. Write the equation of a line that is perpendicular to line m and passes through the point $(6, -1)$.
- 9-32. Thui made the following hypotheses: $2n - 1 < 5$ and $n + 1 \leq 2n$. Which of the following conclusions can she make?
- a. $1 \leq n \leq 3$
 - b. $1 \leq n < 3$
 - c. $1 < n \leq 3$
 - d. $1 < n < 3$
- 9-33. **Multiple Choice:** Which of the expressions below is a factor of $6m^2 + 7m - 5$?
- a. $2m + 1$
 - b. $m + 5$
 - c. $2m - 5$
 - d. $3m + 5$

9.2.2 What if the inequality is not linear?

Graphing Linear and Non-Linear Inequalities



In Lesson 9.2.1, you discovered that the solutions of a linear inequality with two variables can be represented by a shaded region on one side of the line. But how can the graph of an inequality help solve a problem? And what happens when the inequality is not linear? Consider these questions as you complete the following problems with your study team.

- 9-34. Review what you learned about graphing inequalities in Lesson 9.2.1 by graphing the inequality below on graph paper.

$$y \geq -\frac{5}{3}x - 3$$

- What is the minimum number of points you need to test in order to know which side of the line the solution falls on?
- Orville thinks that using the point $(0, 0)$ to test this inequality is a great idea. Why is using this point so convenient?
- Anita decided to use the point $(-3, 2)$ to test the inequality. Test the inequality with her point. Does this point help her decide which side to shade? Why or why not?

- 9-35. FOREIGN AID

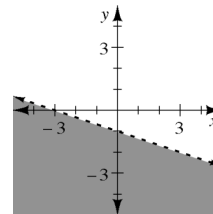
One of the purposes of the United Nations is to have nations work together to help each other. Recently, the members of the U. N. decided to give grants to poor countries to help reduce poverty. However, the United Nations only has the resources to help those countries in the greatest need. Therefore, it was decided that only countries in which the number of people in poverty is **more than** one-half of its total population would receive foreign aid.



- Write an inequality that represents the criteria to receive foreign aid. Let x represent the population and y represent the number of people in poverty.
 - On the Lesson 9.2.2 Resource Page provided by your teacher (also available at www.cpm.org), find the graph that shows the number of people in poverty per the population for each of the countries being considered for foreign aid. Carefully graph your inequality from part (a) on this data graph. Which countries should receive foreign aid?
- 9-36. What if an inequality is non-linear? Decide with your team how to graph the inequality $y < x^2 - 4x + 3$ on graph paper. Your graphing shortcuts can help.

9-37. With your team, graph the following inequalities on graph paper.

a. $y < -\frac{2}{3}x + 4$ b. $y \geq x^2$ c. $x < 2$



9-38. Write the inequality for the solution graphed at right. Be prepared to explain how you found your rule.

MATH NOTES

METHODS AND MEANINGS

Solving One-Variable Inequalities

To solve a one-variable inequality, first treat the problem as if it were an equality. The solution to the equality is called the **boundary point**. For example, $x = 12$ is the boundary point for the inequality $10 - 2(x - 3) \geq -8$, as shown below.

<p>Problem: $10 - 2(x - 3) \geq -8$</p> <p>First change the problem to an equality and solve for x:</p>	$10 - 2(x - 3) = -8$ $10 - 2x + 6 = -8$ $-2x + 16 = -8$ $-2x = -24$ $x = 12$
---	--

Since the original inequality is true when $x = 12$, place your boundary point on the number line as a solid point. Then test one value on either side in the *original* inequality to determine which set of numbers makes the inequality true. Therefore, the solution is $x \leq 12$.

<p>When the inequality is $<$ or $>$, the boundary point is <i>not</i> included in the answer. On a number line, this would be indicated with an open circle at the boundary point.</p>	<table style="width: 100%; margin-top: 10px;"> <tr> <td style="width: 50%;">Test: $x = 8$</td> <td style="width: 50%;">Test: $x = 15$</td> </tr> <tr> <td>$10 - 2(8 - 3) \geq -8$</td> <td>$10 - 2(15 - 3) \geq -8$</td> </tr> <tr> <td>$10 - 2(5) \geq -8$</td> <td>$10 - 2(12) \geq -8$</td> </tr> <tr> <td>$0 \geq -8$</td> <td>$-14 \geq 17$</td> </tr> <tr> <td style="text-align: center;">TRUE!</td> <td style="text-align: center;">FALSE!</td> </tr> </table>	Test: $x = 8$	Test: $x = 15$	$10 - 2(8 - 3) \geq -8$	$10 - 2(15 - 3) \geq -8$	$10 - 2(5) \geq -8$	$10 - 2(12) \geq -8$	$0 \geq -8$	$-14 \geq 17$	TRUE!	FALSE!
Test: $x = 8$	Test: $x = 15$										
$10 - 2(8 - 3) \geq -8$	$10 - 2(15 - 3) \geq -8$										
$10 - 2(5) \geq -8$	$10 - 2(12) \geq -8$										
$0 \geq -8$	$-14 \geq 17$										
TRUE!	FALSE!										



9-39. **Multiple Choice:** Which of the expressions below is a factor of $6x^2 + 7x - 20$?

- a. $3x - 4$ b. $2x - 5$ c. $3x + 4$ d. $4x - 3$

9-40. **Multiple Choice:** Which of the following expressions is the product of $(4y - 3x)(2y + x)$?

- a. $8y^2 - 2xy - 3x^2$ b. $6y^2 - 2xy - 2x^2$
c. $8y^2 + 10xy - 3x^2$ d. $6y^2 - 2x$

9-41. **WHAT'S THE DIFFERENCE?**

Examine the following situations in which you need to find the difference between two amounts.

- a. Rocio has \$298 saved in the bank, while Thomas has \$314. What is the difference between their bank balances? How did you get your answer?
- b. The temperature in Minneapolis on January 10 ranged between -23° and 19° Fahrenheit. What was the difference between the high and low temperatures for this date? How did you get your answer?
- c. Urban High School has 1850 students, while Metro High School has 1490 students. What is the difference of their student populations?
- d. Explain why these differences in (a) through (c) are all positive.

9-42. Solve the following equations and inequalities for x . Check your solution(s), if possible.

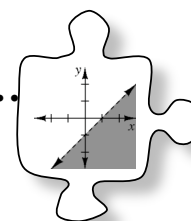
- a. $\frac{3}{x} = 9$ b. $\sqrt{x} = 4$ c. $x^2 = 25$ d. $2(x - 3) > 4$

9-43. During a race, Bernie ran 9 meters every 4 seconds, while Barnaby ran 2 meters every second and got a 10-meter head start. If the race was 70 meters long, did Bernie ever catch up with Barnaby? If so, when? **Justify** your answer.

9-44. Find the x -intercepts of the graph of $y = 5x^2 + 7x - 6$ using *two different methods*. The answers from each method should match.

9.2.3 What's the difference?

Introduction to Absolute Value



In the past few lessons, you learned what inequalities are and learned how to graph linear and non-linear inequalities with two variables. Today you will learn a new operation and will learn how you can use it to create new and interesting inequality graphs.

9-45. ABSOLUTE-VALUE OPERATION

Your teacher will present you with information about an operation called **absolute value**. As your teacher finds the absolute values of numbers such as -11 and 4 , record the results on your paper. Look for a pattern!

- Study the relationship between the number entered in the parentheses and the results shown. Write a statement describing this operation.
- Why would you ever need an absolute value?

abs(-11)	11
abs(4)	4
abs(-100)	100

9-46. While some graphing calculators, like the one shown in problem 9-45, display the absolute value as “ $\text{abs}(-100) = 100$,” the written notation is $|-100| = 100$. This notation consists of two vertical lines on each side of the input value.

- The expression $|-3| + 1$ can be translated as, “Change -3 to a positive value and then add 1.” Translate the expression $|-5 + 1| - 3$ into words and then find its value.
- Evaluate these expressions:
 - $|-100| - 98$
 - $5|2 - 8|$
 - $|-13| + |0|$
 - $14 - |-10 + 3|$
- Now create your own expression using the absolute value that has a result of 10. Be creative and be ready to share your expression with the class.


9-47. Mr. Guo is thinking of a number. When he takes the absolute value of his number, he gets 15. What could his number be? Is there more than one possible answer?

9-48. Riley wants to know what an absolute value might look like on a graph.

- Set up a table and graph $y = |x|$.
- Describe for Riley what the graph looks like. Be as detailed as you can.

- 9-49. Dorinae is confused. She is making a table for $y = |x + 1|$. She is trying to find y when $x = -3$, but she is not sure if she should find the absolute value first, or if she should first add 1. Explain to Dorinae what she should do first. **Justify** your reasoning.
- 9-50. Graph the inequality $y < |2x - 1|$. Be ready to share your graph with the class.

MATH NOTES



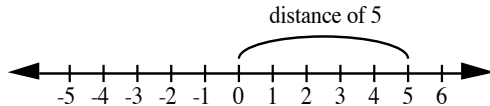
METHODS AND MEANINGS

Definition of Absolute Value

An **absolute value**, represented by two vertical bars, $| \ |$, determines the positive value of a number. Numerically, it represents a distance on a number line between a number and zero. Since a distance is always positive, the absolute value is *always* either a positive value or zero. The absolute value of a number is *never* negative.

For example, the number -3 is 3 units away from 0, as shown on the number line at right. Therefore, the absolute value of -3 is 3. This is written $|-3| = 3$.

Likewise, the number 5 is 5 units away from 0. The absolute value of 5 is 5, written $|5| = 5$.





- 9-51. Brazil's rain forests currently cover about 1,400,000 square miles, but are becoming smaller every year because of deforestation. Realizing that the rain forests are a great resource, Brazil has decided to control how quickly the forests are cleared.
- In 50 years Brazil would like the rain forests to cover more than 1,200,000 square miles. If x represents the forest area that is cleared each year, write and solve an inequality that would help determine acceptable values of x .



9-52. Calculate the value of each expression below. You may want to refer to the Math Notes box for this lesson for help.

a. $|-4|-3$ b. $|6-11+3|$ c. $-9-|-2|$ d. $5|6|-2$

9-53. Clifford thinks that $x = 7$ is a solution to $3(x - 2) \leq 4$. Is he correct? Show why or why not.

9-54. Graph the inequalities below on graph paper.

a. $y \leq -x + 5$ b. $y > \frac{2}{3}x - 1$

9-55. Zachary has \$718 in his bank account and automatically withdraws (subtracts) \$14 every month to pay for his computer service. Christian has \$212 in his bank account and deposits (adds) \$32 each month from his newspaper-delivery tips. Assuming they make no other deposits or withdrawals, when will Zachary and Christian have the same amount of money in their bank accounts?

9-56. Stacey is the star of the basketball team. She makes many baskets during each game and could break the record for the most baskets made in one season at her high school. The data for the first five games of this season is below.

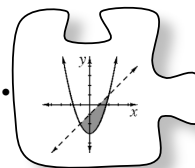
<u>Game Number</u>	<u>Total Number of Baskets</u>
1	6
2	11
3	18
4	25
5	31



- Plot a graph with these data points.
- Draw a trend line for this data using two carefully selected points that best represent the data.
- Use the equation of your line to predict how many baskets Stacey will make by the end of the season if the season has 15 games.

9.3.1 How can I represent it?

Systems of Inequalities



In Chapter 6 you learned that the solution to a system of equations is a point that makes both equations true. But what about the solution of a system of two inequalities? How can you represent these solutions on a graph? How many solutions can a system of inequalities have?

Consider these questions as you learn how to graph a system of inequalities.

9-57. Find your graphs for problem 9-54.

- Compare your solution graphs for $y \leq -x + 5$ and $y > \frac{2}{3}x - 1$ with those of your teammates. Correct any errors. Be sure to focus on whether the boundary line should be included in each graph.
- What would the graph of the system of inequalities look like? Consider the system of inequalities below. Which points are solutions to this system (that is, which points make *both* inequalities true)?

$$y \leq -x + 5$$

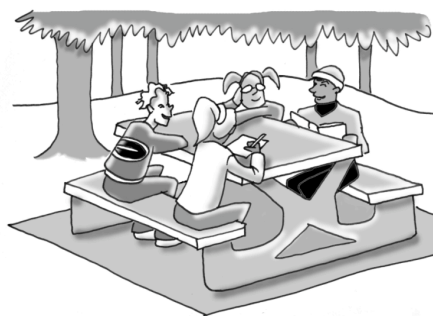
$$y > \frac{2}{3}x - 1$$

- If you have not done so already, verify your solution region from part (b) algebraically by substituting the coordinates of a point from your solution region into each inequality.
- How can you be sure that this region is the only set of points that makes both inequalities true?

9-58. Draw a graph of the region satisfying both inequalities at right. Start by graphing the boundary lines and then test points to find the region that makes both inequalities true.

$$y < x + 2$$

$$y \leq 10 - \frac{3}{4}x$$

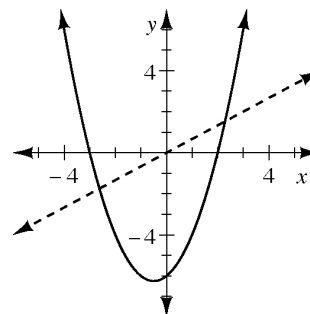


9-59. HOW MANY REGIONS?

When graphing the system of inequalities below, Reyna started with the boundary graph of each inequality shown at right.

$$y \leq x^2 + x - 6$$

$$y > \frac{2}{3}x$$



- Why is the line dashed while the parabola is not?
- Find a copy of Reyna's graph on the Lesson 9.3.1B Resource Page provided by your teacher. How many possible solution regions are there? Carefully count each region with your teammates.
- Pick a point in each region and test it in the system of inequalities. Shade any regions that contain solutions to both inequalities. How many regions make up the solution to this system?
- Why is $(0, 0)$ not a good point to use to test for this solution?

9-60. How does changing the inequality affect the solution graph? Notice that each system of inequalities below uses the same boundary graphs as Reyna's graph from problem 9-59. However, notice that this time the inequalities are slightly altered.

With your teammates, devise a method to determine which region (or regions) are solutions for each system. Shade the appropriate regions on your resource page.

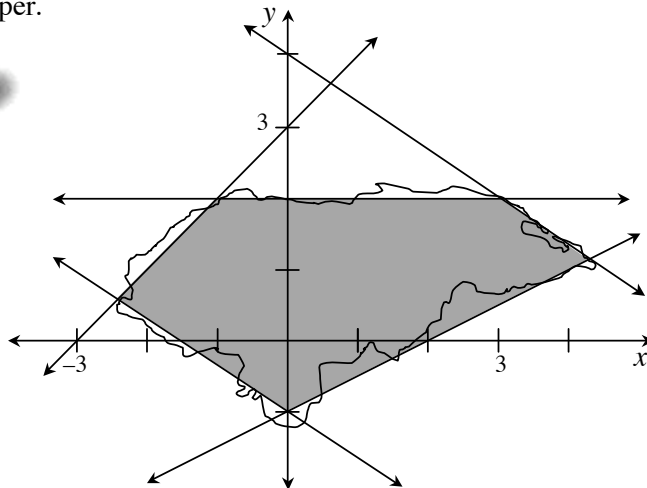
- | | | | | | |
|----|--|----|--|----|--|
| a. | $y \geq x^2 + x - 6$
$y > \frac{2}{3}x$ | b. | $y \geq x^2 + x - 6$
$y < \frac{2}{3}x$ | c. | $y \leq x^2 + x - 6$
$y < \frac{2}{3}x$ |
|----|--|----|--|----|--|

- 9-61. The United Nations asked every nation to write a system of inequalities that best approximates its country's shape (the U.N. thinks this will help find each country's area). Honduras sent in its inequalities by fax, but some of the information is unreadable. With your study team, determine the missing parts of the inequalities and rewrite them on your paper.

$$y \leq x + 3 \quad y \geq \frac{1}{2}x - 2$$

$$y \leq 2 \quad y \leq -\frac{2}{3}x + 4$$

$$y \leq -\frac{2}{3}x - 1$$



MATH NOTES

METHODS AND MEANINGS

Graphing Inequalities with Two Variables

To graph an inequality with two variables, first graph the boundary line or curve. If the inequality does not include an equality (that is, if it is $>$ or $<$ rather than \geq or \leq), then the graph of the boundary is dashed to indicate that it is not included in the solution. Otherwise, the boundary is a solid line or curve.

Once the boundary is graphed, choose a point that does not lie on the boundary to test in the inequality. If that point makes the inequality true, then the entire region in which that point lies is a solution. Examine the two examples below.

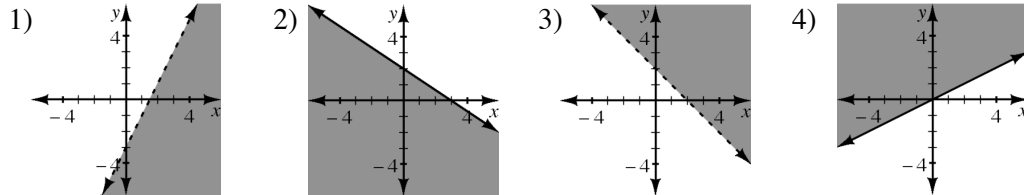
$y < -\frac{2}{3}x + 2$

$y \geq x^2 - 1$



9-62. Match each graph below with the correct inequality.

- a. $y > -x + 2$ b. $y < 2x - 3$ c. $y \geq \frac{1}{2}x$ d. $y \leq -\frac{2}{3}x + 2$



9-63. Solve each inequality below. Represent the solutions on number lines.

- a. $7x - 2 < 3 + 2x$ b. $\frac{1}{3}x \geq 2$
 c. $3(2m - 1) - 5m \leq -1$ d. $2k + 3 \leq 2k + 1$

9-64. Which of the following expressions are equal to 1? (Note: More than one expression may be equal to 1!)

- a. $\frac{114}{114}$ b. $\frac{2}{3} \cdot \frac{3}{2}$ c. $\frac{m+4}{m+4}$ d. $\frac{p^2}{p \cdot p}$

9-65. Factor the following quadratics completely.

- a. $5x^2 + 13x - 6$ b. $6t^2 - 26t + 8$ c. $6x^2 - 24$

9-66. When a family with two adults and three children bought tickets for a movie, they paid a total of \$27.75. The next family in line, with two children and three adults, paid \$32.25 for the same movie. Find the adult and child ticket prices by writing a system of equations with two variables.

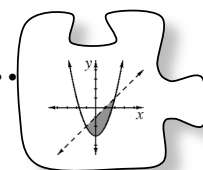


9-67. **Multiple Choice:** Which of the points below is a solution of $y < |x - 3|$?

- a. (2, 1) b. (-4, 5) c. (-2, 8) d. (0, 3)

9.3.2 How can I apply it?

More Systems of Inequalities



- 9-68. Review what you learned about systems of inequalities in Lesson 9.3.1 by graphing the system of inequalities at right on graph paper. Carefully shade the region of points that make *both* inequalities true.

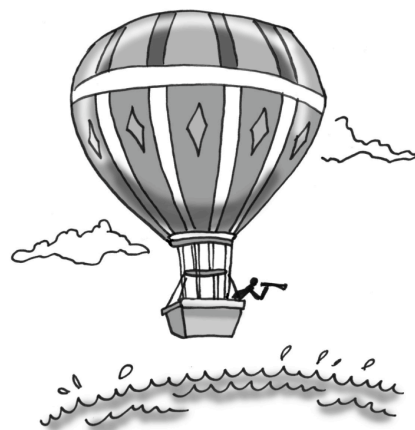
$$y \leq |x| + 4$$

$$-x + 4y \geq 4$$

- 9-69. SEARCH AND RESCUE

“I’m completely lost... water everywhere I can see... both engines have failed... Wait! I see land. I’m going to try to land. I think it’s...”

Those were the last words heard from Harold in his hot-air balloon. The last time the balloon showed up on radar, it was near the Solomon Islands in the Pacific Ocean.



Your Task: Your team must determine where to send the search-and-rescue teams! Use the following reports along with the map on the Lesson 9.3.2 Resource Page (also available at www.cpm.org) and look carefully for information that will help you draw boundary lines. Write a system of inequalities to give to the search-and-rescue team. Be sure to identify the probable landing site on the map.

Basic facts of the case:

The balloon departed from the airport at the very northern tip of the Philippines. The flight was supposed to follow a straight path *directly* to an airport in French Polynesia.

The balloon’s last known location was at $(-1000, 1000)$ near the Solomon Islands.

Pilot’s report from a nearby airplane:

“We were on our way from Australia, when we saw a hot-air balloon sinking rapidly. I am certain that it crashed south of our flight path. When we left Australia, we traveled 2000 km north for every 3000 km east that we flew.”

Phone call received today:

“I was a passenger on a flight that flew directly from French Polynesia to Indonesia. I was looking out my window when I saw the balloon going down to the north of where we were flying.”

9-70. Notice that each system of inequalities below contains the same boundary lines. On graph paper, graph the boundaries for the system on one set of axes. Then, for each pair of inequalities, work with your teammates to decide which region is the solution, if a solution exists. Be ready to share your conclusions with the class.

a. $y \leq \frac{2}{3}x + 3$
 $y \geq \frac{2}{3}x$

b. $y \leq \frac{2}{3}x + 3$
 $y \leq \frac{2}{3}x$

c. $y \geq \frac{2}{3}x + 3$
 $y \leq \frac{2}{3}x$



9-71. In your Learning Log, describe your method for graphing systems of inequalities for a student who has missed class for the last couple of days. Be sure to include examples and important details. Title this entry “Graphing Systems of Inequalities” and include today’s date.



9-72. Graph and shade the solution for the inequality below.

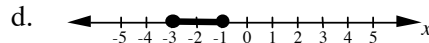
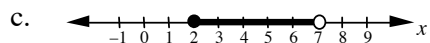
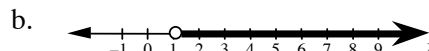
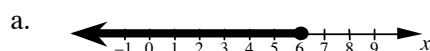
$$y \leq x^2 + 2x - 8$$

9-73. Graph and shade the solution for the system of inequalities below.

$$y \geq \frac{3}{4}x - 2$$

$$y < -\frac{1}{2}x + 3$$

9-74. Write the inequality that represents the x -values highlighted on each number line below.



9-75. Determine if the following statements are true or false.

a. $|-6| < 4$ b. $|-3+5| > 2.5$ c. $4 \geq |0|$ d. $|-4+3| > 1$

9-76. **Multiple Choice:** Which equation below is perpendicular to the line $y = \frac{1}{3}x + 7$?

- a. $x + 3y = 4$ b. $x - 3y = 4$ c. $3x + y = 4$ d. $3x - y = 4$

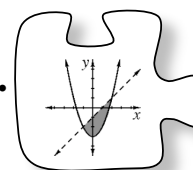
9-77. For the Spring Festival, the Math Club is selling rulers for \$1 and compasses for \$2.50.



- a. While the club would like to sell as many items as they can to raise funds, they need to make at least \$15.00 to break even. Write an inequality to represent this situation. Let r = the number of rulers sold and c = the number of compasses sold.
- b. School rules state that the club can sell a maximum of 25 items for the festival. Write an inequality for this constraint (limitation).
- c. Graph the inequalities from parts (a) and (b) on the same set of axes so that compasses are represented on the x -axis and rulers are represented on the y -axis. Find the region of points that are solutions to each of them. Can this region fall below the x -axis or to the left of the y -axis? Why or why not?
- d. What do the points in the solution region represent?

9.3.3 How can I use inequalities to solve problems?

..... Applying Inequalities to Solve Problems



Today you will pull together all of the mathematics you have studied in this chapter and **apply** it to solve an application problem.

9-78. UNITED NATIONS TO THE RESCUE

As a representative of your country, you have been sent the following letter and given an important task:

Dear Representative to the United Nations:

A critical matter has come to the attention of the United Nations. In the past, when a catastrophe struck a part of the world, the U. N. gathered supplies to give to people in need. Unfortunately, because the U. N. had to collect supplies from each country at the time of the catastrophe, it was always quite a few days before the supplies could be sent to the areas that needed them the most.

A recommendation has come before the U. N. to create a supply of food and medicine packages for future emergencies. Each food package will be able to feed several hundred people, while each medicine package will supply one first-aid station. I am asking each country to donate the same number of packages so each country shares the burden equally.

I am asking each country to determine how many food and medicine packages they are able to give. You will present your findings at today's United Nations meeting. Please be certain to use the information that your country's Budget Committee has prepared to help you decide how many packages you can afford.

Best of luck, and may our efforts make our world a better place!

*Sincerely,
The Secretary General of the United Nations*

After consulting with your country's Budget Committee, your teacher will supply you with some information that will help decide how many food and medicine packets your country can afford.

Your Task: To communicate your country's budget constraints, write an inequality expressing how many food and medicine packages your country is able to give. Let x equal the number of food packages and y equal the number of medicine packages.

On the Lesson 9.3.3B Resource Page provided by your teacher (also available at www.cpm.org), graph the solution region representing the number of medicine and food packets that can be donated by your country. Be prepared to share your graph with the other countries of the United Nations.



- 9-79. As a member of the United Nations, you must consider each of the following proposals. In each case, assume that the United Nations would like to receive as many emergency supplies as possible, while still having each nation give equally.
- One proposal is that each country gives 185 medicine packages. How many food packages should the United Nations require from each country in this case? Explain how you made your decision.
 - Another proposal is to get the largest number of medicine packages possible. What is the largest number of medicine packages that each country can offer? How did you find your answer?

9-80. EXTENSION

A last-minute proposal suggests balancing the number of food and medicine supplies. For instance, if a country gives 150 food packages, then they would also give 150 medicine packages. How many food and medicine packages should the United Nations require from each country in this case? Explain how you determined your solution.



- 9-81. While setting up a mathematical sentence to solve a problem, Paulina and Aliya came up with the equations below. Since the equations did not look alike, the girls turned to you for help.

Paulina: $4x + 2y = 6$

Aliya: $12x + 6y = 18$

- Are these equations equivalent? That is, will the graph of each line be the same? Explain how you know.
- Find another equation that is equivalent to both of these. How did you find your equation?

- 9-82. The town you live in has decided to limit the amount of trash thrown out each month. Your town, which has 3280 homes, has asked each household to keep track of how many pounds of trash they produce during a month. In addition, the town council has found that other sources of trash, such as local businesses, combine to create 1500 lbs of trash each month. If the town has a goal of creating **less than** 50,000 lbs of trash, how much trash should a household be limited to? Write an inequality for this situation and solve it.



- 9-83. Solve the following inequalities for the given variable and represent the solutions on a number line.
- | | |
|---------------------------------|-------------------------------|
| a. $2 < 2m - 8$ | b. $\frac{1}{3}x - 1 \leq -3$ |
| c. $5(2x - 8) + 24 > 3(4 + 2x)$ | d. $5 + 2k < k - 2 + k$ |

- 9-84. Graph the system of inequalities below.

$$\begin{aligned} y &\geq x(x - 4) \\ y &< x \end{aligned}$$

- a. Carefully shade the solution region.
- b. Is $(0, 0)$ a solution to this system? How can you tell?
- 9-85. Solve the quadratic equation below *twice*, once using the Zero Product Property and once using the Quadratic Formula. Verify that the solutions from both methods are the same.

$$2x^2 - 19x + 9 = 0$$

- 9-86. Read the following problem. Then decide which system of equations below can represent this situation.

Multiple Choice: The length of a rectangle is 4 units longer than twice its width. If the area is 126 square units, find the length and width.

- | | | | |
|-------------------------------|----------------------------------|----------------------------------|-------------------------------|
| a. $w = 2l + 4$
$wl = 126$ | b. $l = 2w + 4$
$l + w = 126$ | c. $w = 2l + 4$
$l + w = 126$ | d. $l = 2w + 4$
$wl = 126$ |
|-------------------------------|----------------------------------|----------------------------------|-------------------------------|

Chapter 9 Closure What have I learned?

Reflection and Synthesis

The activities below offer you a chance to reflect on what you have learned during this chapter. As you work, look for concepts that you feel very comfortable with, ideas that you would like to learn more about, and topics you need more help with. Look for **connections** between ideas as well as **connections** with material you learned previously.

① TEAM BRAINSTORM

With your team, brainstorm a list for each of the following topics. Be as detailed as you can. How long can you make your list? Challenge yourselves. Be prepared to share your team's ideas with the class.



Topics: What have you studied in this chapter? What ideas and words were important in what you learned? Remember to be as detailed as you can.

Ways of Thinking: What Ways of Thinking did you use in this chapter? When did you use them?

Connections: What topics, ideas, and words that you learned *before* this chapter are **connected** to the new ideas in this chapter? Again, make your list as long as you can.

② MAKING CONNECTIONS

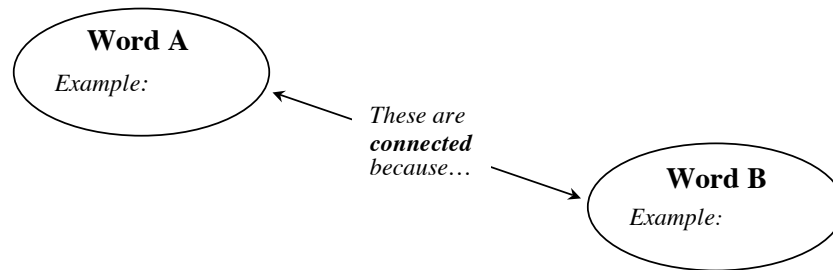
The following is a list of the vocabulary used in this chapter. The words that appear in bold are new to this chapter. Make sure that you are familiar with all of these words and know what they mean. Refer to the glossary or index for any words that you do not yet understand.

absolute value	boundary	coordinates
equation	graph	inequality
number line	region	solution
system of inequalities		

Make a concept map showing all of the **connections** you can find among the key words and ideas listed above. To show a **connection** between two words, draw a line between them and explain the **connection**, as shown in the example on the following page. A word can be **connected** to any other word as long as there is a **justified connection**. For each key word or idea, provide a sketch that illustrates the idea (see the example on the following page).

Continues on next page →

② *Continued from previous page.*



Your teacher may provide you with vocabulary cards to help you get started. If you use the cards to plan your concept map, be sure either to re-draw your concept map on your paper or to glue the vocabulary cards to a poster with all of the **connections** explained for others to see and understand.

While you are making your map, your team may think of related words or ideas that are not listed here. Be sure to include these ideas on your concept map.

③ **SUMMARIZING MY UNDERSTANDING**

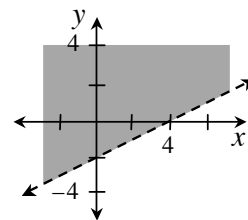
This section gives you the opportunity to show what you know about certain math topics or ideas. Your teacher will give you directions for exactly how to do this.

④ **WHAT HAVE I LEARNED?**

This section will help you evaluate which types of problems you have seen with which you feel comfortable and those with which you need more help. This section appears at the end of every chapter to help you check your understanding. Even if your teacher does not assign this section, it is a good idea to try the problems and find out for yourself what you know and what you need to work on.

Solve each problem as completely as you can. The table at the end of the closure section has answers to these problems. It also tells you where you can find additional help and practice on problems like these.

CL 9-87. Write an inequality that represents the graph at right.



CL 9-88. Find the equation of the line that passes through the points $(-3, 13)$ and $(4, -1)$.

CL 9-89. Is the point $(0, 4)$ a solution to the system of inequalities at right? **Justify** your answer.

$$y \leq -3x + 4$$

$$y > x^2 + 3x - 2$$

CL 9-90. Factor these quadratic expressions completely, if possible.

a. $x^2 + x - 30$

b. $-3x^2 + 23x - 14$

c. $2x^2 - 5x + 4$

d. $6x^2 + 10x - 24$

CL 9-91. Solve each inequality below for the given variable. Then represent each solution on a number line.

a. $4x - 3 \geq 9$

b. $3(t + 4) < 5$

c. $\frac{2y}{7} < 8$

d. $5x + 4 > -3(x - 8)$

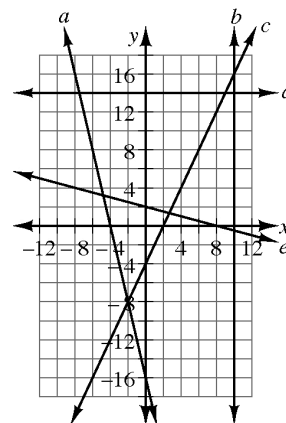
CL 9-92. Brian was holding a ballroom dance. He wanted to make sure girls would come, so he charged boys \$5 to get in but girls only \$3. The 45 people who came paid a total of \$175. How many girls came to the dance?

CL 9-93. Solve these quadratic equations using any method.

a. $0 = 3x^2 + 4x - 7$

b. $x^2 - 3x + 18 = 0$

CL 9-94. Write equations for lines (a) through (e) shown in the graph at right.



CL 9-95. Graph the system of inequalities below on graph paper.

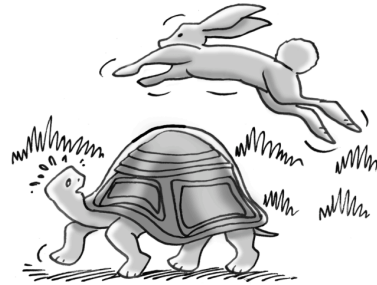
$$y < x^2$$

$$y \geq x + 2$$

CL 9-96. Lew says to his granddaughter Audrey, “Even if you tripled your age and added 9, you still wouldn’t be as old as I am.” Lew is 60 years old. Write and solve an inequality to determine the possible ages Audrey could be.

CL 9-97. Graph the solutions of $y \geq |x + 2|$ on graph paper.

CL 9-98. The hare leaps 500 centimeters every 20 seconds. The tortoise crawls 250 centimeters every 50 seconds, but gets a 1000-centimeter head start. Use any method you know to determine how long it takes the hare to catch up to the tortoise.



CL 9-99. Check your answers using the table at the end of the closure section. Which problems do you feel confident about? Which problems were hard? Use the table to make a list of topics you need help on and a list of topics you need to practice more.

⑤ HOW AM I THINKING?

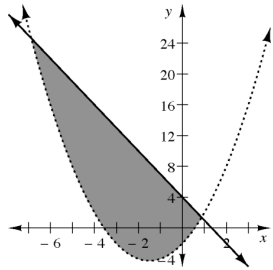
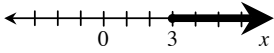
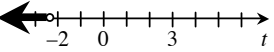
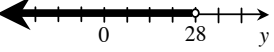
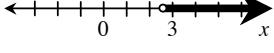
This course focuses on five different **Ways of Thinking**: reversing thinking, justifying, generalizing, making connections, and applying and extending understanding. These are some of the ways in which you think while trying to make sense of a concept or to solve a problem (even outside of math class). During this chapter, you have probably used each Way of Thinking multiple times without even realizing it!

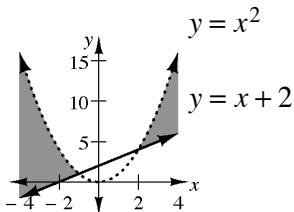
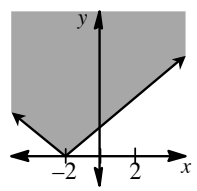


Review each of the Ways of Thinking that are described in the closure sections of Chapters 1 through 5. Then choose three of these Ways of Thinking that you remember using while working in this chapter. For each Way of Thinking that you choose, show and explain where you used it and how you used it. Describe why thinking in this way helped you solve a particular problem or understand something new. (For instance, explain why you wanted to **generalize** in this particular case, or why it was useful to see particular **connections**.) Be sure to include examples to demonstrate your thinking.

Answers and Support for Closure Activity #4

What Have I Learned?

Problem	Solution	Need Help?	More Practice
CL 9-87.	$y > \frac{1}{2}x - 2$	Lesson 9.3.1 Math Notes box	Problems 9-38 and 9-62
CL 9-88.	$y = -2x + 7$	Lesson 7.3.3 and Lesson 7.3.4 Math Notes box	Problems 9-19 and 9-31
CL 9-89.	Yes; the point (0, 4) lies on the graph $x^2 + 3x - 2 < y \leq -3x + 4$. Therefore, it is a solution to the system of inequalities.	Problems 9-57 and 9-59	Problems 9-58, 9-60, 9-68, 9-70, 9-73, and 9-84
			
CL 9-90.	a. $(x+6)(x-5)$ b. $(-3x+2)(x-7)$ c. not factorable d. $2(x+3)(3x-4)$	Problems 8-13, 8-14, and 8-35; Lesson 8.1.4 Math Notes box	Problems 9-22, 9-33, 9-39, and 9-65
CL 9-91.	a. $x \geq 3$  b. $t < -\frac{7}{3}$  c. $y < 28$  d. $x > 2.5$ 	Lesson 9.2.2 Math Notes box	Problems 9-6, 9-8, 9-14, 9-17, 9-28, 9-63, and 9-83
CL 9-92.	25 girls came to the dance.	Lesson 7.1.4 Math Notes box	Problems 9-7, 9-30, 9-66, and 9-86

Problem	Solution	Need Help?	More Practice
CL 9-93.	a. $x = 1$ or $x = -\frac{7}{3}$ b. no solution	Lessons 8.1.4, 8.2.3, 8.3.1, and 8.3.2	Problems 9-9, 9-21, and 9-85
CL 9-94.	a. $y = -4x - 16$ b. $x = 10$ c. $y = 2x - 4$ d. $y = 14$ e. $y = -\frac{1}{4}x + 2$	Lesson 7.2.2 Math Notes box	Problems 9-18, 9-19, and 9-31
CL 9-95.		Problems 9-57 and 9-59	Problems 9-58, 9-60, 9-68, 9-70, 9-73, and 9-84
CL 9-96.	Audrey is less than 17 years old.	Lessons 9.1.1 and 9.2.2 Math Notes boxes	Problems 9-29, 9-51, 9-77, and 9-82
CL 9-97.		Lessons 9.2.3 and 9.3.1 Math Notes boxes	Problems 9-25, 9-26, 9-50, 9-54, 9-62, 9-62, and 9-72
CL 9-98.	The hare catches up to the tortoise after 50 seconds.	Lesson 7.1.4 Math Notes box	Problems 9-43 and 9-55